

# European Yield Differentials and Basis Risk: A Hedging Perspective<sup>\*</sup>

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## ABSTRACT

Hedging and speculative motives of market makers often require strategies involving positions in both the futures and spot market. These are basis strategies. The risk involved in the payoff of these strategies depends on the basis risk and cannot be diversified away. Hence, any security that contains a higher basis risk should be compensated through a higher yield. Using simulations based on a risk-averse model, we find that a market maker increases his quoted spread modestly when basis risk increases. However, if the basis risk becomes very large, the quoted spread increases more than proportionally. This convexity in spread suggests the following: the market maker increases his spread as a compensation for the increased hedge difficulty. When the basis volatility becomes very large however, the quoted spread becomes even larger indicating his unwillingness to trade. Based on these findings, we study the basis risk for four fixed income securities in Europe. Using a state space approach applied to trading data, we estimate the basis risk volatility and find that bonds that are traded at a premium, like Germany and France, have a lower basis volatility. This gives an additional explanation why some European bonds are traded at a significant premium even when we take differences in liquidity or credit risk into account.

Keywords: Bonds markets, Microstructure, Yield Differentials, Hedging

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## ABSTRACT

Hedging and speculative motives of market makers often require strategies involving positions in both the futures and spot market. These are basis strategies. The risk involved in the payoff of these strategies depends on the basis risk and cannot be diversified away. Hence, any security that contains a higher basis risk should be compensated through a higher yield. Using simulations based on a risk-averse model, we find that a market maker increases his quoted spread modestly when basis risk increases. However, if the basis risk becomes very large, the quoted spread increases more than proportionally. This convexity in spread suggests the following: the market maker increases his spread as a compensation for the increased hedge difficulty. When the basis volatility becomes very large however, the quoted spread becomes even larger indicating his unwillingness to trade. Based on these findings, we study the basis risk for four fixed income securities in Europe. Using a state space approach applied to trading data, we estimate the basis risk volatility and find that bonds that are traded at a premium, like Germany and France, have a lower basis volatility. This gives an additional explanation why some European bonds are traded at a significant premium even when we take differences in liquidity or credit risk into account.

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# 1 Introduction

This paper analyzes the impact of basis volatility on the pricing of Eurozone sovereign bonds. Hedging and speculative motives of market makers often require strategies involving positions in both the futures and spot-market. These are so-called basis strategies and arise from the market makers need to hedge incoming order flow. The payoff of these strategies depends on the basis volatility or basis risk. In this paper we argue that basis risk is a relevant factor in determining the price of a fixed income security.

This paper can be divided in two parts. The first part of the paper analyzes the impact of order flow on price formation when hedging is involved. We show that the basis risk (or basis volatility) is relevant as it determines the final payoff of a traders hedged position. Basis risk is also important due to costs associated with managing inventory and holding a position.<sup>1</sup> These costs are mostly operational costs, waiting costs or a deviation from an optimal (mean-variance efficient) portfolio. The more difficult it is to hedge, i.e. the larger the basis volatility, the more difficult it is to manage these costs and the higher the required compensation for offering immediacy. Using simulations we find that there exist a convex relation between the quoted spread and basis risk. More specific, a widening in the quoted spread occurs due to a small increase in the basis risk. The quoted spread however widens even more when basis risk becomes large. This non-linearity in spread dynamics suggests the following: a market maker requires a higher compensation for his services when his exposure to basis risk increases by quoting a larger bid-ask spread. When the basis risk becomes problematic however, he widens his spread even more and this indicates his reluctance to trade. In the second part of the paper, we estimate the basis risk for some Eurozone government bonds using transaction data from the MTS trading system and bund future data from the EUREX. Although the bund future requires the delivery of German bonds, there exist a relation between the futures and the cash market even if the cash instruments cannot be delivered because the futures and spot market are driven by the same interest rates. If we exclude the futures market in our analysis, there still exist a hedging relationship between sovereign bonds through the repomarket.<sup>2</sup> We show that bonds with a larger basis volatility are traded at a premium. This provides an alternative explanation for the yield differences in the Eurozone besides credit risk or liquidity premium. As

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<sup>1</sup>See for example Garman (1976), Stoll (1978), Ho and Stoll (1981) or O'Hara (1995) for a more general overview.

<sup>2</sup>To see this, consider a market maker buying a Belgian treasury bond. In a back-to-back trade this security could be sold right away to another customer, earning the bid-ask spread. However, it is most likely that this bond is not sold immediately, and the market maker must raise funds to finance this position through a repo. Besides the financing of this trade, the market maker must also hedge its new position by selling a similar security with the same duration. In this case, the market maker will conduct a reverse repo financed by the cash from the short sale. Duffie (1996) addresses the issue of special repo rates in a theoretical framework. He finds that anyone who holds a special security can borrow at low costs and reinvest this in the general collateral repo rate, earning a repo investment. As a result, securities which are on special (or likely to go on special) will carry a higher price than otherwise identical issues. Jordan and Jordan (1997) provide empirical evidence supporting this idea and find that the liquidity premium associated with "on-the-run" issues is likely due to repo specialness.

far as we know, we are the first to address the differences in yields towards hedging quality using trading data. This paper does not address the determinants of basis volatility. On an intra-day basis, the level of liquidity is an important factor of the basis volatility. Consequently, one cannot disentangle the yield spread into a liquidity component and a basis risk component. Moreover, any econometric model including both a measure of liquidity and basis risk will face severe forms of multi-collinearity.

Most papers analyze the determinants of yield differentials from the perspective of country specific factors, default risk or liquidity. Chakravarty and Sarkar(1999) focuses on the bid-ask spread in the US bond market. The data is comprised by individual insurance companies from 1 January 1995 until 31 December 1997 with daily trading volume as a proxy for liquidity. They find that the bid-ask spreads in municipal and corporate bonds are comparable with an average spread of 22 and 21 cents respectively. The treasury bond spread however is significant lower with an average spread of 11 cents. This smaller spread in government securities is explained in terms of a default and liquidity premia. For Eurozone bonds, Bernoth, Hagen and Schuknecht (2003) state that *"the main analytical problem is whether these interest differentials can be explained by default risk and or liquidity risk premia."* Differences in credit rating exist among European sovereign bonds running from a "AAA " status in Germany, France, Austria and the Netherlands to A+ for Greek bonds.<sup>3</sup> In a small paper by the central bank of Norway, it is shown that (in the pre-Euro area) the yield differentials are a function of the country specific macroeconomic variables like government debt, budget deficit, current account and credit rating.<sup>4</sup> The importance of credit spread has been shown by Codogno, Favero and Missale (2003) for the Euro area. They note that *"the risk of default is a small but important component of yield differentials."* and *"liquidity factors play a smaller role."* Many studies also argue that illiquid fixed income securities should provide a higher yield in order to induce investors to keep these securities in their portfolio. See for example Amihud and Mendelsohn (1991), Warga (1992) and Strebulaev (2001). In the Eurozone, the operations conducted by the treasury agents in the primary market for 10-year bonds vary from EUR 5bn in Finland to 15-25bn in Germany and Italy. Differences also exist in the secondary market. In terms of activity, trading in the Italian and German securities are among the largest in the world. Cheung, de Jong and Rindi (2004) provide some information about the trading activity on the MTS trading system which is the largest interdealer trading platform for European government bonds. They find that some 85% of all trading activity in the running 10-year bonds stems from trading in Italian BTP securities.

Interestingly, the liquidity and trading activity story does not fully explain the yield differences in the Eurozone. To see this, take a look at figure 1. This figure shows the yield spread between various 10-year benchmark bonds issued in 2003 in the Eurozone. As we can see, the Dutch state

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<sup>3</sup>Based on S&P's credit rating system.

<sup>4</sup>Paper was written by Ingunn Lønning: "Default premia on European Government debt." Year of publication unknown.

loan is traded on average at a lower yield than its French equivalent (average spread is -2.2 bp) while the Netherlands is less active on the issuance side. The result is even more interesting for the Italian security, which is the most actively traded security on the secondary market. It is however trading above its Portuguese ‘equivalent’ (average spread is 7.1bp) and traded at par with the Greek bond (average spread is -0.4bp). Figure 2 gives us a snapshot from the tradeweb platform<sup>5</sup> of prices quoted by market makers on the European bond market. Let us take a look at the depicted quote for the same Italian 2013 bond. This security is very liquid due to its supply in the primary market and its activity in the secondary market but traded at a 13.4bp yield pickup compared to its German ‘equivalent’. On the other hand, the credit ratings tell a different story. The Portuguese and Italian securities are equal in credit risk (according to Moody’s and S&P) while their Greek ‘equivalent’ is at one notch below.<sup>6</sup> The Portuguese bonds however are trading at a yield pickup of some 4bp pickup while Greece is trading almost at par with the Italian 2013 bond. However, the trading activity in the secondary market for these securities is merely a fraction of the activity found for Italian bonds. Favero, Pagano and von Thadden (2004) focuses solely on liquidity and default risk on Eurozone government bonds but they agree that these two factors are not able to explain the differences in yields. The importance of hedging using the bund future is also recognized:

“(....) bonds traded in the cash market are not considered as a perfect hedge for position in the bund future.”

The fact that yield differences are not only related to liquidity or credit risk but also to hedging quality has important implications for policymaking. A strong fiscal convergence and operations leading to an increase in liquidity are important for convergence of bond yields but any measures that can limit the basis risk should be taken into consideration as well. This can be achieved by e.g. cash settlement or allowing non-German bonds for delivery. Although these measures do not solve additional problems<sup>7</sup>, it would help in lowering the ‘natural’ advantage incorporated in German bonds due to their physical delivery. We think that these measures can greatly improve the efficiency of using futures to hedge Eurozone fixed income securities.

The paper is organized as follows. Section 2 provides an introduction of basis risk and its role in hedging and speculative strategies. Section 3 outlines the cost-of-carry relation between the bund future and the spot market securities and gives a description of our dataset. Section 4 introduces the model including some econometric details and provides a discussion of the estimation results. Section 5 finally concludes.

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<sup>5</sup>Tradeweb is the largest client to dealer trading platform for European bonds.

<sup>6</sup>Portuguese and Italian securities have a credit rating of Aa2 (Moody’s) or AA (S&P) while Greece trades one notch below.

<sup>7</sup>Like cash constraints during a roll-over period or the existence of additional conversions for the delivery of non-German securities.

## 2 Hedging European Sovereigns

Transactions in futures are usually either outright or against (forward) bond positions in the form of basis trades. The basis  $B_{i,t}$  represents a combination of the futures contract and a spot market security. Buying the basis involves the purchase of securities and the simultaneous sell of the futures contracts as given in equation (1) and the exact opposite holds when we sell the basis.

$$B_{i,t} = P_{i,t} - c_i F_t \quad (1)$$

Hull (1997) defines the basis as the difference between the spot price of assets to be hedged and the futures price of the contract. In the case of bond futures however, the basis is defined on a hypothetical bond. We therefore need to multiply the bond future with the conversion factor in order to get the basis. A long line of research has analyzed the concept of the basis risk, mostly in the context of hedging or speculation. For example Working (1953), Ederington (1979), Figlewski (1984), Briys, Crouchy and Schlesinger (1993), Castellino (2000), Mahul (2002) and Draper and Fung (2003). These papers look at the existence of the basis risk, its impact on the hedge quality and the payoff of basis strategies. Grossman (1988) analyzes the informational role of the basis and argues that the basis does not only unify the futures and spot- market but also reflects the different preferences on both markets. We now take a look at the impact of basis risk and the way this affects the investor's position.

### 2.1 The Role of Basis Risk in Hedging and Speculation

Let us consider a short hedger<sup>8</sup> with a portfolio consisting of a long position of  $X_B$  units of an asset with price  $P_t$  and a short position of  $\theta c X_B$  futures contracts. The market maker is free to choose  $\theta > 0$ . The  $(T - t)$  profit of his portfolio equals  $X_B (P_T - P_t) - \theta c X_B (F_T - F_t)$  with variance  $X_B^2 \sigma_B^2 + \theta^2 c^2 X_B^2 \sigma_F^2 + 2\theta c X_B \sigma_{BF}$ . Without loss of generality, we can normalize  $X_B = 1$  and express the expected profit and variance in terms of the basis per unit of his spot market position. The profit and its variance equals

$$\begin{aligned} E(\pi^S) &= (P_T - \theta c F_T) - (P_t - \theta c F_t) \\ &= E\Delta B_T + (1 - \theta) c \times E\Delta F_T \end{aligned} \quad (2)$$

$$\sigma_\pi^2 = \sigma_B^2 + [(1 - \theta) c]^2 \sigma_F^2 + 2(1 - \theta) c \sigma_{BF} \quad (3)$$

where  $\Delta B_T = B_T - B_t$  and  $\Delta F_T = F_T - F_t$ . Although we focus on the role of basis risk as a measure for hedging quality, it is also of importance for investors who are using the basis for speculative purposes. Working (1953) already questioned the view of hedgers being risk minimizers and emphasized expected profit maximization. He states

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<sup>8</sup>A short hedger takes long positions in the basis. This requires a short position in futures and a long position in cash bonds.

*”(The hedger) buys the spot commodity because the spot is relative low to the futures price, (...), therefore he buys spot and sells the future.”*

Also trading the basis occurs in conjunction with other commitments

*“ usually in the expectation of a favorable change in the relation between the spot and futures price.”*

Working’s statement is being captured by equation (2) and shows us that the expected profit in a hedge consist of two components:

1. Change in the expected basis component  $E\Delta B_T$ . If a strengthening of the basis occurs (i.e.  $B_T$  widens as the bond price increases more than the futures price), the short hedgers position improves while a weakening results in a worsening of his position.
2. Change in a speculative component  $(1 - \theta) c \times E\Delta F_T$  which is a function of his control variable  $\theta$ . If futures prices are unbiased, i.e.  $E_t F_{t+1} = F_t$ , than the expected hedge profit is only affected by the expected change in the basis  $E\Delta B_{t+1}$ .

Hence, a holder of a long position in the cash market will (over) hedge if the basis is expected to increase and (under) hedge if the basis is expected to fall. Figlewski (1984) argues that basis is the risk which arises as the connection between the futures market and the underlying is imperfect. For bond futures, a perfect hedge ( $B_t = 0$ ) arises if bond  $i$  is the (hypothetical) bond specified in the futures contract. Any other bond being delivered will have  $B_t > 0$  to avoid arbitrage opportunities.<sup>9</sup> Equation (3) tells us that the risk involved in a short hedge is an increasing function of the basis risk  $\sigma_B$  which cannot be controlled for by choosing an appropriate  $\theta$ . By minimizing (3) with respect to  $\theta$  we find the minimum variance hedge (MVH):

$$MVH = \min_{\theta} \sigma_{\pi}^2 \Rightarrow \theta_{\min} = 1 + \rho_{BF} \frac{1}{c} \frac{\sigma_B}{\sigma_F} \quad (4)$$

Again, the number of contracts that one chooses is a function if the basis risk  $\sigma_B$ . By substituting equation (4) into the variance gives us the residual risk:

$$\sigma_{\pi}^2(\theta_{\min}) = \sigma_B^2 (1 - \rho_{BF}^2) \quad (5)$$

Equation (5) shows that the residual risk depends on the basis risk  $\sigma_B^2$  but also on the correlation between the basis and futures contract  $\rho_{BF}$ . More important, the residual risk will always exist unless the basis and the futures contract is perfect correlated. the results show that the basis

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<sup>9</sup>To see this, equation (1) can be interpreted as the cash flow for a strategy with immediate delivery of the cash bond. The expected cash flow that arises equals  $c_i F_t - P_{i,t} = -B_{i,t}$  implying  $b_t \geq 0$  to avoid arbitrage. In here, the trading of the basis often involves repos as both the long and short need funds to pay (obtain) for the purchase (delivery) of the securities.

risk influences the hedge quality and the payoff of a speculative strategy. Any investor, who is conducting a basis strategy, should take the basis risk into account.

Our next focus is the analysis of a dealer's quoted spread when he faces basis risk. One can argue that a market maker can control the basis risk by moving the quoted spot price in line with the futures contract. However, the dealer is limited for a number of reasons: First, according to Chan (1992), movements in the futures price are a source market wide information while movements in the spot instruments are a source of individual news. Hence, the dealer must take individual news into account. In addition, a change in the dealer's quoting strategy will induce orders that are not of the dealer's preference as it will result in a change of the trader's (optimal) position of wealth and inventory. Also, the urgency to hedge depends also on the market makers ability to match (unwanted) incoming buy or sell orders with incoming sell or buy orders. The volatility of order flow is therefore also of importance.

## 2.2 Basis Risk and Quoted Spread

In this section, we consider the impact of the basis on the quoted spread. In order to analyze this, we propose a two-period model involving a risk averse market maker. The following assumptions are made:

- The trading period is given  $\Theta_1 = [t_0, \tau)$  where at time  $t_0$  the market makers decides his bid-price  $E(P_T) - b$  and his ask-price by  $E(P_T) + a$ . No trades arrive in period  $\Theta_2 = [\tau, T]$ . The second period can be interpreted as the after market hedging period in which the market maker can decide to hedge the position acquired in period  $[t_0, \tau)$ .
- We have one single market maker who is free to quote his bid-ask spread and no entrance by another market maker is possible in the period considered. Hence, there is no strategic interaction or interdealer trading possible.
- The market maker is only quoting one security and any hedging must go through the futures contract rather than an offsetting position in another security through the repomarket. At any time  $t \in [t_0, T]$ , this portfolio consist of a trading account ( $Y$ ), a position in the security ( $X$ ) and a position in the hedge instrument ( $Z$ ).
- The price of this hedging instrument is exogenous, i.e. cannot be influenced by the market maker himself. This means that the futures market is infinitely large and elastic compared to the spot market. We assume that market making is his only stream of income.
- Denote  $P_t$  as the price of the security with a conversion factor  $c$  relative to a futures contract with price  $F_t$ . The futures price follows a random walk, i.e.  $F_t = F_{t-1} + \varepsilon_{t,f}$  where  $\varepsilon_{t,f} \sim$



$N(0, \sigma_f^2)$ . Both securities are correlated through

$$P_t = B_t + cF_t \quad (6)$$

$$B_t = \beta B_{t-1} + \varepsilon_{t,b} \quad (7)$$

where  $\varepsilon_{t,b} \sim N(0, \sigma_b^2)$  and  $\sigma_b$  denoting the basis risk. The innovations  $\varepsilon_{t,f}$  and  $\varepsilon_{t,b}$  are allowed to be correlated.

- Let  $Q^A$  and  $Q^B$  be the total number of buy and sell orders that the market maker is receiving in period  $\Theta_1$ . Both are functions of the bid and ask price set by the market maker.<sup>10</sup>

$$Q^A = \xi_a a^{-1} + \varepsilon_{sell} \quad (8)$$

$$Q^B = \xi_b b^{-1} + \varepsilon_{buy}$$

where  $\varepsilon_{i(i=buy, sell)} \sim N(0, \sigma_i^2)$  distributed.

- Let  $h$  describes the hedge ratio of his position and  $H \equiv h(Q^b - Q^a)$  is the fraction of his position which is hedged using a futures contract. The decision to hedge is made at time  $\tau$ .
- Assume that the market maker is risk averse and maximizes the utility of terminal wealth. The market maker optimizes

$$\max_{S=a+b, h} U = \max_{S=a+b} E(W_T) - \frac{1}{2}\gamma\sigma_W^2 \quad (9)$$

$$\text{subject to } EW_T \geq 0 \text{ and } S \geq 0 \quad (10)$$

where  $\gamma$  is his constant relative risk aversion. Equations (10) tells us that a market maker will stop quoting when he expects to end with a negative wealth.

**Step 1:** To solve the optimization problem, we work backwards by first solving the problem of the optimal hedge fraction. Terminal wealth  $W_T$  can be expressed as a function of the value of his inventory  $X_T$ , a hedge position  $Z_T$  and a trading account  $Y_T$ :

$$W_T = X_T + Z_T + Y_T$$

$$X_T = (X_{t_0} + Q^b - Q^a) P_T$$

$$Z_T = h(Q^b - Q^a) \Delta F_T$$

$$Y_T = [Y_{t_0} + Q^a(p+a) - Q^B(p-b)](1+r)^{T-\tau}$$

The time  $T$  value of the dealer's inventory equals his initial position  $X_{t_0}$  plus the number of securities bought ( $Q^b$ ) minus the number of securities sold ( $Q^a$ ). The inventory is evaluated at price  $P_T$ . We assume that  $X_{t_0} = 0$  and the inventory at the end of period  $\Theta_2$  arises solely from

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<sup>10</sup>From the perspective of the dealer.

his market making activity in period  $\Theta_1$ . This assumption implies (i)  $Z_T$  does not contain any futures position prior to period  $\Theta_1$  and (ii)  $Z_T$  is the change in the margin account. The trading account is given by his initial size  $Y_{t_0}$  plus the value  $Q^a$  sold at the ask-price  $(p + a)$  minus the value  $Q^B$  bought at the bid-price  $(p - b)$ . Any borrowing and lending occurs at a rate  $r$  and is constant between period 0 and  $T$ . We come to the following proposition:

**Proposition 1** *The fraction of the inventory being hedged by the dealer equals*

$$H^* \equiv h^* (Q^b - Q^a) = - \left[ \frac{\text{cov}(B_T, F_T | \Omega_\tau)}{\text{var}(F_T | \Omega_\tau)} + c \right] (Q^b - Q^a) \quad (11)$$

**Proof.** See appendix point 1 ■

Equation (11) shows that an important role is played by the correlation between the basis and the futures price. If the basis is independent of the futures price (i.e.  $\text{cov}(B_T, F_T | \Omega_\tau) = 0$ ), the market maker will take a hedge ratio equal to the conversion factor. For example, if  $Q^b > Q^a$ , the dealer will short sell  $c(Q^b - Q^a)$  futures contracts.<sup>11</sup> Using the notation of the previous section, we must have  $\theta = 1$ , resulting in an expected profit equal to

$$E(\pi^S) = \Delta B_T \quad (12)$$

On the other hand, if  $\text{cov}(B_T, F_T | \Omega_\tau) \neq 0$ , the market maker will short hedge  $-H^*$  futures contracts and this implies  $\theta = \frac{\text{cov}(B_T, F_T | \Omega_\tau)}{\text{var}(F_T | \Omega_\tau) c} + 1$ . In this case, the expected profit equals

$$E(\pi^S) = E\Delta B_T + \frac{\text{cov}(B_T, F_T | \Omega_\tau)}{\text{var}(F_T | \Omega_\tau) c} E\Delta F_T \quad (13)$$

which creates an additional speculative component in the hedge strategy depending on the change in the futures price.

**Step 2:** To see how basis risk influences the quoted price, consider the quoting decision taken at time  $t_0$ . Note that at time  $t_0$  the terminal wealth can be expressed as a function of the decision variables  $(a, b)$  by substituting  $H^*$  as given in equation (11) into the wealth function. The decision of choosing  $(a, b)$  depends on the information set  $\Omega_0 = \{F_0, P_0\}$ . If we denote this function as  $W_T^*$ , the objective function becomes

$$\max_{a,b} E(W_T^* | \Omega_0) - \frac{1}{2} \gamma \text{var}(W_T^* | \Omega_0) \quad (14)$$

Where the components of  $W_T^* = (Y_T + Z_T) + X_T$  are given by

$$\begin{aligned} X_T &= (Q^b - Q^a) P_T \\ Z_T &= -c(Q^b - Q^a) \Delta F_T \\ Y_T &= Y_{t_0}(1+r)^{T-t_0} + [Q^a a - Q^B b] (1+r)^{T-\tau} \end{aligned}$$

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<sup>11</sup>Provided that the numeraire in the futures contract is the same as the spot position. As an illustration, assume for example that  $c = 0.95$  and the total inventory position is EUR 10mio. If the futures contract is specified per EUR 100.000, the dealer will short sell 950 futures contracts for hedging purposes.

The conditional expectation and variance of wealth are given by

$$E(W_T^*|\Omega_0) = \xi(a+b)P_0 + X_{t_0}(1+r)^T - \xi(a^2+b^2)(1+r)^{T-\tau} \quad (15)$$

$$\begin{aligned} var(W_T^*|\Omega_0) = & var(X_T|\Omega_0) + var(Z_T|\Omega_0) \\ & + var(Y_T|\Omega_0) + 2cov(Y_T, X_T|\Omega_0) \end{aligned} \quad (16)$$

Note that  $(Q^b - Q^a) \sim N(\xi(b+a), \sigma_b^2 + \sigma_a^2)$  and the calculation of the conditional variance requires the products of dependent Gaussian random variables for which we need simulations in  $Q^a$  and  $Q^b$  in order to solve the problem.<sup>12</sup>

The expected wealth is not only a function of the basis risk but also depends on the volatility of the market maker's order flow<sup>13</sup>. A basis risk equal to zero implies that the outstanding position is perfectly hedged using  $cF_\tau$  futures. In this case, the only source of uncertainty stems from net order flow uncertainty as it influences the trading account of the dealer. Hence, the quoted spread depends on the basis risk but also on  $\phi \equiv \sigma_{buy}^{-1}\sigma_{sell}$ . The larger  $\phi$  deviates from 1, the more difficult it is to match order flows and the larger the uncertainty in the trading account. Because of this reason, we take the volatility in the buy ( $\sigma_{buy}$ ) and sell order flow ( $\sigma_{sell}$ ) into consideration. The exact simulation process is outlined in the appendix (step 2). The outcome of the simulation process is depicted in figure 3 and shows that the quoted spread is a non-linear increasing function of basis volatility. This non-linearity suggests that the market maker is controlling his exposure to basis risk by increasing his spread as a compensation for the increased hedge difficulty. However, when the basis volatility becomes very large, the quoted spread becomes even larger indicating his reluctance to trade. Note also that a  $\phi$  different from 1 will result in a higher quoted spread in order to limit the exposure of order flow on the trading account.

### 3 Data

In the previous section we saw that the basis risk is important in determining the quality of a hedge and the payoff of any arbitrage strategy. An investor holding a security with a higher basis risk should be compensated through a higher yield. Using this idea, we estimate the basis risk for some 10-year Eurozone government bonds using transaction data from the MTS trading system and the EUREX for the bund future during the period January 2000 to May 2001.

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<sup>12</sup>The problem stems from the fact that terminal wealth is function of order flow  $\times$  price and both are normally distributed. If the random variables  $Q_t$  and  $P_t$  were independent, things would be simplified. A general case about the product of independent Gaussian variables has been given by e.g. Goodman (1960). For an exposition of the probability density function, see Springer and Thompson (1970).

<sup>13</sup>Recall that the urgency to hedge depends on the market makers ability to match (unwanted) incoming buy or sell orders with incoming sell or buy orders.

### 3.1 The Cost of Carry Relation

In the absence of market frictions and uncertainty, the futures price should equal the price of the underlying bond plus the cost of carry. Any breakdown of this relationship must result in an exposure since traders hedge the position in the spot market using an offsetting position in the futures market. A large number of studies focus on the relationship between the spot and derivatives market. See for example, Stoll and Whaley (1990), Chan (1992), Huang and Stoll (1994), Brooks, Garret and Hinich (1999) for stock and stock futures indices and Bhattacharya (1987) and de Jong and Donders (1998) for the stock and stock options market. These studies find significant lead-lag relationships although this relationship is not unidirectional as the cash index may affect the futures market too. The rationale between these relationships is found in market microstructure frictions that break the cost-of carry-relationship and the leverage character of futures markets, which creates better trading opportunities for informed traders. Subrahmanyam (1991) discusses the role of information and shows in a theoretical setup that liquidity traders prefer to trade the basket rather than the underlying security. The reason for this is twofold. First, the transaction cost of basket trading like futures is much lower compared to the individual securities. Second, the security specific adverse selection component tends to diversify away in a basket. However, this increased activity of liquidity trading also facilitates the incoming of informed traders as they can better hide their strategic trades among the noise, see e.g. Kyle (1985).

Let us now turn our attention to the German (Bund) futures contract that is used in this study. This contract is based on a hypothetical bond with a coupon of 6 percent and a maturity of exactly 10 years starting on the settlement date. Quotation in this contract is in a percentage of par value carried out in two decimals. The contract size is EUR 100.000 and every 0.01 percent of price movements represent EUR 10. Delivery of bonds takes place on the 10-th day of March, June, September or December or the immediate following trading day. The last trading day is always two exchange days prior to the delivery day where trading in the running contract stops at 12.30 CET. Trading hours are between 8.00 and 19.00. The daily settlement price is based on the volume weighted average price of the last 5 trades. If these trades are older than 15 minutes or if more than 5 trades occurred during the last minute, all trades during that 15-minute period are considered. The final settlement price is determined at 12.30 p.m. CET on the last trading day and is based on the last ten trades, provided that they are not older than 30 minutes. If more than 10 trades occurred during the last minutes all trades are considered in this period. Although the bund futures contract is based on a hypothetical bond, delivery is based on a tangible asset. In order to avoid any manipulations<sup>14</sup> in the spot market, one can deliver any German bond with

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<sup>14</sup>Because of the relative size of the futures market to the spot market, the futures market is potentially exposed to price manipulations. A strategy often depicted as an example is the short squeeze. Investors can take a long position in the futures contract and the underlying bond. Investors who want to cover their short position will drive up the futures price. At the same time, any investor who wants to deliver the specified contract will drive up the bond price.

a maturity between 8.5 and 10.5 years and a minimum issue size of EUR 2bn. Any bond that satisfies the contract specifications may be delivered, and this will result in an conflict of interest. A holder of a long position hopes to receive a bond with a high coupon and significant accrued interest while a holder of the short position hopes to deliver a bond with a low coupon shortly after the coupon payment date. In order to solve this conflict of interest, the amount exchanged for the bond (invoice price) will be adjusted. Because the investor being short in the futures contract has the delivery option, he will receive an invoice amount at date  $T$  equal to

$$c_i F_T + acc_{i,T} \quad (17)$$

where  $F_T$  is the futures settlement price,  $c_i$  is the conversion factor of bond  $i$  being delivered and  $ACC_{i,T}$  the accrued interest of bond  $i$  at time  $T$ . The conversion factor is simply the price unit of face value such that every bond has the same yield if purchased. The yield selected for calculation is the same as the coupon rate in the definition of the contract. If we denote the coupon of the hypothetical bond by  $\bar{\gamma}$ , we can calculate the conversion factor for bond  $i$  with coupon  $\gamma_i$  maturing at time  $M$  in a discrete setup as<sup>15</sup>

$$\begin{aligned} c_i(T, c_i, M) &= \sum_{i=1}^{M-T} \frac{\gamma_i}{(1 + \bar{\gamma})^i} + \frac{1}{(1 + \bar{\gamma})^{M-T}} \\ &= \frac{\gamma_i}{\bar{\gamma}} + \frac{\bar{\gamma} - \gamma_i}{\bar{\gamma} (1 + \bar{\gamma})^{M-T}} \end{aligned} \quad (18)$$

We see that if  $\gamma_i = \bar{\gamma}$ , the conversion factor equals 1. If  $\gamma_i > \bar{\gamma}$ , the conversion factor is larger than 1 and smaller otherwise. The conversion factor shows us that, by adjusting the price, one can provide any investor with the approximately the same yield with different coupons. However, conversion factors are not a waterproof method when the term structure is not flat at the notional coupon rate. One can show (appendix point 3) that the futures price follows the price of the underlying deliverable bond, its repo rate and the time until maturity. All the parameters needed to calculate the fair price of the futures contracts are known in advance.<sup>16</sup> Benninga and Wiener (1999) show in a continuous setting that the optimal CTD will have either the largest or lowest coupon (given a fixed maturity) as long as there are no delivery options in the contract. Intuitively, given a maturity, the duration of a bond is determined by the coupon and bonds with the highest duration (i.e. lowest coupon) becomes relative cheap compared to other deliverable securities when the curve steepens.

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<sup>15</sup>In our case, we set  $\bar{\gamma} = 6\%$  for the bund futures contract.

<sup>16</sup>Running a cash and carry strategy also involves the paying of a repo rate while at the same time earning interest on the coupons. When the coupon rates are higher than the repo rates, we have a positive carry. As a result, future prices with a longer delivery date have a lower price as it cost less to conduct the strategy. On the other hand, if we have negative carry, the futures price for securities with a longer delivery date is higher.

Table 1: Trading characteristics of 2011 and 2012 securities.

Table shows us some trading characteristics of the bonds considered for our econometric analysis. We focus on the 10-year benchmark bonds of Belgium (OLO), France (OAT), Germany (DBR) and Italy (BTP). E.g. OLO5 09/11 reflect a Belgian bond with a coupon of 5% which matures in September 2011.

Bond Type	OLO5 09/11	OAT5 10/11	DBR5.25 01/11	BTP5.25 08/11
Total number of trades	5542	4754	2886	62735
Percentage EuroMTS trades	32	67	72	21
Total volume	48472	32028	17520	354140
Volume EuroMTS	14585	20875	12905	80758
Average local volume	8.99	7.11	5.71	5.52
Average EuroMTS volume	8.22	6.55	6.21	6.13
EuroMTS / local volume	0.91	0.92	1.09	1.11
percentage 5mio trades	27	67	81	88
percentage 10mio trades	71	32	17	9

### 3.2 Bond Data

The bonds that we use in our analysis are the running benchmark 10-year bonds issued in 2001 by Italy, Belgium, France and Germany. There are two reasons to consider these bonds. First, the number of observations of these bond series is the largest and therefore most suitable for statistical inference. Second, the 10-year area of the European yield-curve is very active in terms of trading activity and issuance by government agents. It is also considered to be the most important long bond on the yield curve. The trading characteristics of these bonds are presented in table 1 which is taken from Cheung, de Jong and Rindi (2004). As we can see, the number of observations is the largest (smallest) for the Italian (German) securities. On the other hand, the average trade size is smallest in the Italian securities. Table 2 uses equation (18) to calculate the conversion factors for the different bonds considered in our analysis

### 3.3 Bund Futures Data

Let us turn our attention to the trading characteristics as found in our dataset. Data of the bund futures contract comes from trading on the EUREX, which is an electronic trading platform for derivatives.<sup>17</sup> The sample of intra-day futures traded on the EUREX system spans the period 2 January 2000 until December 2001. Because our intra-day bond trades run from January 2000 until May 2001, we limit our analysis up to the June 2001 contract. This gives us a total of 6 contracts (4 contracts in 2000 and the March and June, 2001 contract).

Table ?? provides an overview of trading activity per contract as found in our dataset. We see that the largest part of trading activity is concentrated in the front contract. We also present the

<sup>17</sup>An identical contract is traded on the LIFFE in London. A comparable contract is traded on the CBOT. In contrast to its European equivalent, this security has a cash delivery rather than a physical delivery.

Table 2: Conversion factors

Table shows us the conversion factors (assuming that these bonds could be delivered). The conversion factors are calculated using the following formula:

$$CF(T, c, M) = \frac{\gamma_i}{\bar{\gamma}} + \frac{\bar{\gamma} - \gamma_i}{\bar{\gamma}(1 + \bar{\gamma})^{M-T}}$$

where  $\bar{\gamma} = 0.06$  is the coupon of the hypothetical bond,  $T$  is the delivery date and  $M$  the maturity data of the bond.

	OLO5 9/11	OAT5 10/11	DBR5.25 01/11	BTP5.25 08/11
March-02	0.9381	0.9377	0.9545	0.9521
June-02	0.9394	0.9390	0.9555	0.9531
September-02	0.9407	0.9403	0.9565	0.9542
December-02	0.9420	0.9416	0.9576	0.9552
March-03	0.9433	0.9429	0.9586	0.9562
June-03	0.9447	0.9442	0.9597	0.9572

most important results graphically for the June 2001 and September 2001 contract (patterns are similar for other series). Figure 4 depicts the large trading activity in the June 2001 contract until 1<sup>th</sup> June (day 100 in the picture), which is the beginning of the expiration month and 8 days before the final trading day of this contract. In contrast, trading is rather modest in the follow up contract as can be seen in figure 5, but trading activity picks up considerably in the 100-th day while at the same time, the average number of contracts per trades falls. This increased trading activity *in combination with* the small number of contracts per trade suggests that bond traders are actively hedging their portfolio as much as they can. The fact that activity is mainly concentrated in the nearest contract suggests that this is rather a short-term hedge. Active hedging of portfolios is a phenomenon widely seen in these markets. Naik and Yadav (2003) for example provide empirical evidence of bond traders hedging their bond positions using duration measures on the Gilt (UK) market. The analysis shows us that the follow up contracts starts immediately on the first day of the maturity month of the previous contract. Using this information, we construct a single time series of future prices. Every trading day is divided in 96 intervals of 5 minutes and runs from 8.00 and 18.00 CET. We take the volume weighted-price if a trade occurred in these intervals and use *non-available* otherwise.

We follow Bollerslev, Cai and Song (2000) and model futures return as  $\ln F_t - \ln F_{t-1}$  where  $F_t$  is the volume weighted average price in a 5-minute interval. The sample mean of the 5-minute futures series is  $-0.0001$  and indistinguishable from zero at standard significance level giving the sample standard deviation of  $0.027\%$ . However, the returns are clearly not normally distributed. For example, the skewness of  $-0.93$  and a kurtosis of  $66.2$  are both highly significant. At the same time, the maximum and minimum return of  $0.50\%$  and  $0.66\%$  do not represent any sharp discontinuities in the series. In order to evaluate some of the intra-day periodicity of returns, we

Table 3: Some trading characteristics of the bund future

Overview of trading activity on the EUREX for the Bund future as found in our dataset. Values between brackets reflects the observations in the running months. As an illustration, let us take a look at the June 2001 contract. The table shows us that a total of 111 trading days were observed in our dataset giving a total of 720 thousand trades or 42.2 million contracts (this reflects an average of 58.7 contracts per trade). More importantly, trading is concentrated in the period when this contract is front running (1 March until 7 June). During this period, some 710 thousand trades were observed reflecting 41.5 million contracts. This equals 590 thousand contracts per day.

Contract	March 2001	June 2001	September 2001
Number of contracts	33.3	42.2 (41.5)	38.2(37.7)
Number of trades	0.51	0.72 (0.71)	0.64 (0.63)
Average Trading Size	65.5	58.7	60.0
Number of Trading days	49	111	176
Contracts per Day (running months)	0.68	0.59	0.54
Contract	December 2001	March 2002	June 2002
Number of contracts	48.5 (48)	41.7 (41.1)	36.8 (36.4)
Number of trades	0.83 (0.82)	0.85 (0.85)	0.65 (0.64)
Average Trading Size	58.6	48.9	56.8
Number of Trading days	188	179	173
Contracts per Day (running months)	0.70	0.60	0.55

calculate sample mean for the absolute returns. Its pattern is depicted in figure 6 and reflects a broad U-shape which is closely linked to the cycle of market activity. Volatility is gradually increasing until 10.00 CET (interval 24) after which volatility drops with a low around the lunch hours. Market volatility is relative quiet but starts picking up around 14.30 CET (interval 74) in which market makers are preparing themselves for the opening of the US markets. These findings are consistent with Ahn, Cai and Cheung (2002) for the Bund future and Scalia (1998) for Italian treasuries and correspond with the macroeconomic announcements in the US, which are regularly scheduled around 8.30 EST. A large number of studies, e.g. Fleming and Remolona (1997, 1999b), Balduzi, Elton and Green (2001) and Bollerslev, Cai and Song (2000) have linked the intra-day volatility of US treasuries with the release of macroeconomic news. Andersen and Bollerslev (1997, 1998) also divide their dataset in 5 minutes to analyze the role of macroeconomic announcements on volatility. Using high-frequency data, they find intra-day volatility being larger than daily variation in absolute return. In addition, the effect of macroeconomic announcements is strong but short-lived. Ahn, Cai and Cheung (2002) provide a detailed study of the impact of various macroeconomic variables on the Bund future. They find the largest impact for the German IFO industry survey, the industrial production and the Bundesbank policy meetings.<sup>18</sup>

<sup>18</sup>The US numbers are also important. The authors find that especially the NAPM and the unemployment figures have a significant impact.



## 4 Estimating the Basis Risk

Let us now define the econometric model. Denote the price of bond  $i$  at time  $t$  by  $p_{i,t}$  and the unobserved efficient futures price by  $F_t^*$ . In a multivariate setup where we have  $N$  bonds and 1 futures contract, the model is given by

$$\mathbf{p}_t = \hat{\mathbf{b}}_t + \mathbf{c}F_t^* \quad (19)$$

where the parameters in boldface denote vectors and in capital boldface denote matrices. Here  $F_t^*$  is the (unobservable) efficient price. So far no residual term is introduced because equation (19) reflects a exact relation between the price of a security and the efficient price. We follow Hasbrouck (1993) and use the following dynamic structure for the futures price

$$\begin{aligned} F_t &= F_t^* + e_t \\ F_t^* &= F_{t-1}^* + \kappa_t \end{aligned} \quad (20)$$

The futures price equals its efficient price  $F_t^*$  plus some measurement error  $e_t$  while the efficient price follows a random walk with innovation  $\kappa_t$ . The dynamics of the basis for bond  $i$  is given by  $\hat{\mathbf{b}}_t = \mathbf{b} + \mathbf{b}_t$  where  $\mathbf{b}$  is the long run average and  $\mathbf{b}_t$  follows an AR(1) process

$$\mathbf{b}_t = \mathbf{G}\mathbf{b}_{t-1} + \mathbf{r}\kappa_t + \mathbf{u}_t \quad (21)$$

Let us consider the specification of the basis dynamics as given by (21). We know that the residual risk depends not only on the basis risk, but also on the correlation between the futures contract and basis. We therefore include  $\mathbf{r}\kappa_t$  to model the interaction between the basis and the futures contract. In addition, dynamics in the futures contract depends also on shocks in the cash market, which is denoted by  $\mathbf{u}_t$ . Using these ingredients, we write the full model as

$$\begin{aligned} F_t &= F_t^* + e_t \\ \mathbf{p}_t &= \mathbf{b} + \mathbf{b}_t + \mathbf{c}F_t^* \\ F_t^* &= F_{t-1}^* + \kappa_t \\ \mathbf{b}_t &= \mathbf{G}\mathbf{b}_{t-1} + \mathbf{r}\kappa_t + \mathbf{u}_t \\ E(\mathbf{u}_t\mathbf{u}_t') &= \Xi_u, \text{var}(\kappa_t) = \sigma_\kappa^2 \end{aligned} \quad (22)$$

where  $\mathbf{G}$  is a  $N \times N$  diagonal matrix with  $\mathbf{G}_{ii} = \gamma_i$ .<sup>19</sup> If the eigenvalues of  $\mathbf{G}$  lies outside the unit circle, these shocks accumulate over time. Clearly, if  $\gamma_i$  is small, the impacts of previous shocks die out relative fast. If  $\kappa_t$  is uncorrelated with  $\mathbf{u}_t$ , the unconditional covariance matrix of the basis ( $\Xi_{basis}$ ) is given by

$$\text{vec}[\Xi_{basis}] = (\mathbf{I}_{N^2} - (\mathbf{G}' \otimes \mathbf{G}))^{-1} (\text{vec}[\mathbf{r}' \otimes \mathbf{r}] \sigma_\kappa^2 + \text{vec}[\Xi_u]) \quad (23)$$

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<sup>19</sup>Note that  $\gamma_i$  in this section is different than in the previous section (where  $\gamma_i$  reflected the coupon of bond  $i$ ).

provided that  $\gamma_i = 1$  is not an eigenvalue of  $\mathbf{G}$ . Hence, the  $(i, j)^{th}$  element of  $\Xi_{basis}$  can be written as

$$\Xi_{basis}^{(i,j)} = \begin{cases} \frac{r_i r_j \sigma_\kappa^2 + \sigma_{ui}^2}{1 - \gamma_i \gamma_j} & \text{if } i = j \\ \frac{r_i r_j \sigma_\kappa^2}{1 - \gamma_i \gamma_j} & \text{if } i \neq j \end{cases} \quad (24)$$

Now we are ready to construct our hypotheses and relate these hypotheses to testable restriction in our model:

- If bond  $i$  is traded at a premium (in terms of yields) compared to bond  $j$  does it also have a higher basis risk? Specifically, we have to check whether

$$H_0 : \Xi_{basis}^{(i,i)} < \Xi_{basis}^{(j,j)} \quad (25)$$

The structure of the model also enables us to detect the impact of any unexpected news on the basis volatility. Chan (1992) argues that movements in the futures contract are a source of market wide information while movements in the spot instruments are a source of individual news. We therefore make a distinction between the following cases:

- Type 1 news affects the cost-of carry relation through the bund future as it enters the system through the parameter  $\kappa_t$ .<sup>20</sup> If we denote  $\Xi_r = \mathbf{r}' \mathbf{r} \sigma_\kappa^2$ , we can calculate the fraction of this impact on total basis volatility which is given by the square root of the diagonal elements of  $v_1$ :

$$v_1 = \Xi_r \Xi_{basis}^{-1} \quad (26)$$

- Type 2 news affects the cost-of carry relation through the spot market as it enters the system through the vector  $\mathbf{u}_t$ . This is bond specific news. One can think e.g. of shocks due to change in supply or buy-back operations announced by the treasury agent. Because we are working with transaction data, this news must find its way through order flows in the secondary market. If this news is positive (e.g. the treasury is planning to issue less bonds due to a lower state deficit), then these bonds will outperform the rest of the market. The fraction of this news on the total basis volatility is shown by the square root of the diagonal elements of  $v_2$ :

$$v_2 = \Xi_u \Xi_{basis}^{-1} \quad (27)$$

Note that the model as depicted by equation (22) is a reminiscent of Hasbroucks unobserved component model in a multivariate setting. If  $\mathbf{G} = \mathbf{0}$  and  $\mathbf{c} = \mathbf{1}$ , we get the model proposed by de

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<sup>20</sup>One can think for example of technical problems on the EUREX system

Jong and Schotman (2003) for price discovery of securities in a multiple market setting. We use the Kalman filter to estimate the model. One question that may arise is the use of a state space approach for estimating the model. The Bund futures market is one of the most efficient markets in the world and the gain by working with the efficient futures price  $F_t^*$  rather than observable  $F_t$  is probably negligible. Therefore we can also run a simple regression of  $\mathbf{p}_t$  on  $F_t$ . However, because we are working with transaction data, there may exist noise due to the bid-ask spread set by dealers. Moreover, the exact relationship between the fair futures price and the cheapest to deliver bond as given by equation (40) exist through a cash and carry strategy. But from the economic point of view this makes sense only if the underlying bonds are deliverable. Finally, there is also a problem of non-synchronous trading as the activity in the futures market is by far larger than the cash market. This creates problems in terms of spurious correlation which is handled easily by the Kalman filter.

#### 4.1 The Kalman Filter

We briefly discuss the specific estimation procedure for our model. The reader is referred to Harvey (1993, chapter 4), Hamilton (1994, chapter 13) or Durbin and Koopman (2001) for a detailed description of the Kalman filter together with its applications in econometrics. In our case, the Kalman filter is easy to setup as the structure of our model is a reminiscent of a local level model. Let  $f_t$  be the optimal estimator of  $F_t^*$  based on all information up to  $\mathbf{p}_t$  with an associated mean square error  $\phi_t \equiv E[F_t^* - E(f_t)]^2$ . Consider the Kalman filter procedure for time  $t$  to time  $(t+1)$  where  $f_t$  and  $\phi_t$  are given. Based on the equations of (22) we have

$$\begin{aligned} E_t(f_{t+1}) &= f_t \\ E_t(\mathbf{p}_{t+1}) &= \mathbf{b} + E_t(\mathbf{b}_{t+1}) + \mathbf{c}E_t(f_{t+1}) \\ E_t(\phi_{t+1}) &= \phi_t + \sigma_k^2 \end{aligned} \tag{28}$$

with prediction error

$$\begin{aligned} \mathbf{M}_{t+1} &= \mathbf{p}_{t+1} - E_t(\mathbf{p}_{t+1}) \\ &= \mathbf{r}k_{t+1} + \mathbf{u}_{t+1} + \mathbf{c}[F_{t+1} - E_t(f_{t+1})] \end{aligned} \tag{29}$$

Using this we can find the conditional MSE of  $E_t(\mathbf{x}_{t+1})$ :

$$\begin{aligned} \mathbf{F}_{t+1} &\equiv E_t(\mathbf{M}_{t+1}\mathbf{M}_{t+1}') \\ &= \mathbf{r}\mathbf{r}'\sigma_k^2 + \Xi_u + \mathbf{c}E_t[F_{t+1} - E_t(f_{t+1})]\mathbf{c}' \end{aligned} \tag{30}$$

Because we assume that all the parameters are normally distributed, both (28) and (30) are normally distributed and we update the state using

$$f_{t+1} = E_t(f_{t+1}) + E_t(\phi_{t+1})\boldsymbol{\iota}_N'\mathbf{F}_{t+1}^{-1}(\mathbf{p}_{t+1} - \mathbf{b} - \mathbf{G}\mathbf{b}_{t-1} - \mathbf{c}E_t(f_{t+1})) \tag{31}$$

$$\phi_{t+1} = E_t(\phi_{t+1}) - E_t(\phi_{t+1})\boldsymbol{\iota}_N'\mathbf{F}_{t+1}^{-1}E_t(\phi_{t+1})\boldsymbol{\iota}_N \tag{32}$$

Equation (28) to (32) constitutes the Kalman filter which we apply in combination with a maximum likelihood procedure. An important issue here are the initial values  $f_0$  and  $\phi_0$  in order to start the procedure for time  $t = 0$  to time  $t = 1$ . Harvey (1993) argues that if the state process was stationary, one can use the unconditional mean to start the procedure. In our case however,  $F_t^*$  follows a random walk while  $\mathbf{b}_t$  is a stationary process when the eigenvalues of  $\mathbf{G}$  are inside the unit circle. We expect the efficient price to be closely related to the observable futures price, i.e.  $F_0^* = F_0$  and  $\text{var}(F_0^*) = 1000$  to address the uncertainty. In addition, the deviation from the basis is initialized at  $\mathbf{b}_0 = \mathbf{0}$  with an uncertainty of  $\text{var}(b_t) = 100$ .

## 4.2 Missing Observations

High frequency time series are typically not observed in regular intervals. This is clearly the case for our dataset where the futures are traded more often than any of the bonds. In our case, 3 different situations may arise for interval  $t$ . The first situation arises when none elements of  $\mathbf{x}_t$  are missing and the estimation proceeds in its usual way. The second situation arises when all elements of  $\mathbf{x}_t$  are missing. In that particular case, we set  $\mathbf{x}_{t+1} - \beta - \gamma E_t(f_{t+1}) = 0$  and  $E_t(\phi_{t+1}) \iota_N' \mathbf{F}_{t+1}^{-1} = 0$  and the updating process becomes

$$f_{t+1} = E_t(f_{t+1}) \quad (33)$$

$$\phi_{t+1} = E_t(\phi_{t+1}) \quad (34)$$

The third and most occurring situation arises when some, but not all of the elements of the observation vector  $\mathbf{x}_t$  are missing. In this case, we can construct a matrix  $\mathbf{W}$  whose rows are a subset of the rows of the unit matrix  $I_N$  and create a new observation vector  $\mathbf{x}_t^* = \mathbf{W}\mathbf{x}_t$ . The updating procedure proceed exactly the same as in the first situation where at the appropriate time points  $\mathbf{x}_t$  is replaced by  $\mathbf{x}_t^*$  changing the dimension of the observation vector.

The state space procedure is the same as the approach taken by Lo and MacKinlay (1990).<sup>21</sup> They argue that many securities respond to the same news due to common factor components. The fact that some securities are traded less frequently means that these securities responds with a lag, inducing spurious correlation between the closing price of securities, even if the underlying (true) returns are uncorrelated. Lo and MacKinlay (1990) proposes a model for non-synchronous trading by assuming that the true return generating process  $\tilde{R}_t$  is a multifactor model with both common and idiosyncratic factors. The latter ones are assumed to be uncorrelated amongst the  $N$  securities.<sup>22</sup> For every security  $i$  ( $i = 1, \dots, N$ ) they construct a random  $(1 \times T)$  vector  $\mathbf{v}$  such that  $\mathbf{v}_i(t) = 1$  when security  $i$  has been traded in interval  $t$  and 0 otherwise, they relate the observed

<sup>21</sup>There is however a difference in approach. Lo and Mackinlay explicitly model return  $r_t$  and this requires observations  $t$  and  $t - 1$ . As we model prices, only observation  $t$  is required.

<sup>22</sup>De Jong and Nijman (1995) generalizes the approach of Lo and Mackinlay by assuming that the true return generating process may be correlated.

process with the true process by

$$\mathbf{R}_T^{obs} = \mathbf{V}\tilde{\mathbf{R}}_T \quad (35)$$

where  $\tilde{\mathbf{R}}_T = [\tilde{R}_{1t} \dots \tilde{R}_{2t}]'$  and  $\mathbf{V}$  a diagonal matrix with  $\mathbf{v}_i$  on its diagonal. In other words, the observed return is a random sum with a random number of terms. As we can see, the procedure of Lo-MacKinlay is also captured by our state space model where the  $i^{th}$  rows of  $\mathbf{W}$  is a subset of  $\mathbf{v}_i$ .

### 4.3 Estimation Results

Before we start discussing the estimation results, let us consider the quality of the residuals. In order to give an economic interpretation to the model, we must have the residuals being stationary which means that equation (22) incorporates a cointegration relation

$$\mathbf{u}_t = \mathbf{p}_t - \mathbf{c}F_t^* \quad (36)$$

In addition, the underlying assumptions for the Kalman filter are white noise disturbances. On these assumptions, the forecast errors are  $\mathbf{M}_t \sim N(0, \mathbf{F}_t)$  and we have to analyze the forecast errors  $\mathbf{M}_t^* = \mathbf{C}_t \mathbf{M}_t$  where  $\mathbf{C}$  is the Choleski factorization of  $\mathbf{F}_t^{-1}$ . Basic diagnostics are applied to  $\mathbf{M}_t^*$  and a plot of these residuals are given in figure (8). The plots show that that all the series are stationary but they exhibit a strong form of heteroskedasticity. It therefore fails the assumption of white noise underlying the data generating process. The Kalman filter can still be used to calculate the linear projections of  $\mathbf{x}_{t+1}$  on past observations while a likelihood function based on a multivariate Gaussian distribution can be optimized with respect to the unknown parameters. The standard errors however may not be valid and we apply the quasi-maximum likelihood procedure as suggested by White (1982) rather than the usual second order derivatives of the likelihood function.

Table (4) provides some details of the multivariate estimation results for the 2011 bonds. As we can see, the estimated  $\sigma_e$  equals 0.00062 and is highly significant. This shows that the bund futures market is indeed a very efficient market where the observable price is closely related to an efficient price. The average basis  $b_i$  are all positive and smallest for Germany followed Italy, France and Belgium. This is not a surprise as German and Italian securities pay an annual coupon of 5.25% while their French and Belgian equivalent pay 25bp less. The key parameter in our analysis is the variance of the basis which equals  $(\rho_i^2 \sigma_\kappa^2 + \sigma_{ui}^2) (1 - \gamma_i^2)^{-1}$ . As we see,  $\rho_{Italy}$  is not significant. In addition, the values for  $\gamma$  are very close but significantly different from one at the 10% rejection level which means that  $b_{i,t}$  shows signs of a close random walk and indicates a weak reversion back to its long run average. Because  $\gamma_i$  is close to one, we calculate the basis risk not based on the formula but rather take the sample standard deviation of  $\mathbf{b}_t$ . Figure 7 depicts the dynamics of the state variables from the Kalman filter. In here, the FGBL state is the efficient futures price

Table 4: Estimation results for 2011 bonds

The estimated parameters of our state space model for the 2011 bonds with the corresponding standard errors in italics. DBR = German, OAT = France, OLO = Belgium and BTP = Italy. Note that  $\sigma_e$  is multiplied by 1000.

parameters	DBR 2011	OAT 2011	OLO 2011	BTP 2011
$b_i$	1.730 <i>0.969</i>	6.312 <i>0.909</i>	8.584 <i>4.990</i>	3.568 <i>5.340</i>
$c_i$	0.951 <i>0.009</i>	0.870 <i>0.008</i>	0.836 <i>0.046</i>	0.902 <i>0.020</i>
$\rho_i$	0.009 <i>0.003</i>	0.091 <i>0.026</i>	0.091 <i>0.048</i>	<i>0.041</i> <i>0.051</i>
$\gamma_I$	0.971 <i>0.017</i>	0.994 <i>0.001</i>	0.996 <i>0.002</i>	0.999 <i>0.000</i>
$\sigma_e$	6.2 ( $\times 1000$ ) <i>1.13 (<math>\times 1000</math>)</i>			
$\sigma_k$	0.024 <i>0.000</i>			
$\sigma_{ui}$	0.032 <i>0.011</i>	0.012 <i>0.001</i>	0.027 <i>0.008</i>	0.010 <i>0.001</i>
$\sigma_{basis}$	0.16	0.21	0.60	0.64

and closely follows the observable futures price. Using the sample standard deviation, we find a basis volatility running from 0.16 (Germany), 0.19 (France), 0.60 (Belgium) and 0.64 (Italy). Hence, the French and German benchmark bonds have a basis volatility that is more than three times lower than the basis volatility of Belgium and Italy making the uncertainty in the hedge payoff using the bund future much larger in the Belgian and Italian sovereigns. This confirms the hypothesis that bonds with a lower basis risk are traded at a premium. Let us now take a look at the other parameters. We argued that the regression coefficient  $c_i$  could be interpreted as a conversion factor. If we assume that these bonds are deliverable, we can calculate these conversion factors using equation (18). Recall that the true conversion factors are depicted in table (2) and this gives us the opportunity to test whether  $c_i$  equals its conversion factor. In addition, using the values for  $\sigma_{ui}^2$ ,  $\mathbf{r}$  and  $\sigma_\kappa^2$  we are able to calculate the impact of different shocks on the basis volatility. We find that the fraction of shocks in the order flow contributes to some 20% of the basis volatility (Germany), 6.3% (France), 4.3% (Belgium) and 1.5% (Italy). We think that this is an interesting result for the market maker as he has partial control over his exposure to basis volatility by changing the bid-ask spread. In other words, the controllable part of the basis risk is the largest in Germany followed by France. However, some 80% to 98% of the basis volatility is still out of the market makers control.

## 5 Conclusions

In this paper we try to explain the yield differentials in European sovereign bonds from the perspective of the actively traded bund futures contract. Many strategies involve a position in the spot- and futures market and the payoff of these strategies depends on a non-diversifiable basis risk. This paper contributes to the existing literature in two ways. First, using a risk-averse framework and simulations, we find that the quoted spread is a convex function of basis risk. This convexity implies that a dealer will set a higher spread when basis risk increases. If basis risk becomes very large however, the increase in the quoted spread is even larger indicating the market makers reluctance to trade. This convexity in spreads also reflects the importance of the futures market for pricing bonds and explains why trading comes to an halt when the futures market faces trading problems. Second, we show that Eurozone government bonds with higher yields have higher basis volatility. Using a filtering approach in combination with Quasi-Maximum Likelihood we find that the basis risk in German and French securities is much lower than those found for the Belgian and Italian securities. This provides another explanation for the premium observed in these securities even though Italian securities are heavier traded and more liquid in the secondary market. In addition, spot dealers can control a part of their basis risk by setting a price in line with the futures contract. The easier it is to control this basis risk, the less impact high basis volatility has on the market makers position. We find that some 20% of basis risk can be controlled for in German securities while less than 2% of the basis risk can be controlled for in Italian securities when using the bund future.

The fact that price differences are not only related to liquidity or credit risk but also to hedging quality has important implications for policy making. A strong fiscal convergence and operations leading to an increase in liquidity are important for convergence of bond yields but measures that can limit the basis risk should be taken into consideration as well. For the actively traded bund futures contract, this can be achieved by cash settlement or allowing non-German bonds for delivery. Although these measures do not solve additional problems, it would help in lowering the ‘natural’ advantage incorporated in German bonds due to their physical delivery. We think that these measures can greatly improve the efficiency of the Eurozone sovereign bond market.

## Appendix

1. The decision to buy or sell  $h$  number of futures contracts is made at time  $\tau$  when the market maker observes the information set  $\Omega_\tau = \{P_\tau, F_\tau, Q^a, Q^b\}$ . The market makers objective function is given by

$$\max_h EU(W_T|\Omega_\tau) = E(W_T|\Omega_\tau) - \frac{1}{2}\gamma var(W_T|\Omega_\tau) \quad (37)$$

and, in order to find the optimal hedge ratio, we have to calculate its conditional expectation and variance

$$\begin{aligned} E(W_T|\Omega_\tau) &= E(X_T|\Omega_\tau) + E(Z_T|\Omega_\tau) + E(Y_T|\Omega_\tau) \\ E(X_T|\Omega_\tau) &= (Q^b - Q^a) E_\tau(P_T) \\ E(Y_T|\Omega_\tau) &= [Y_{t_0} + Q^a(p+a) - Q^b(p-b)](1+r)^{T-\tau} \end{aligned}$$

$$\begin{aligned} var(W_T|\Omega_\tau) &= var(X_T|\Omega_\tau) + var(Z_T|\Omega_\tau) + var(Y_T|\Omega_\tau) + 2cov(X_T, Z_T|\Omega_\tau) \\ var(X_T|\Omega_\tau) &= (Q^b - Q^a)^2 var(P_T|\Omega_\tau) \\ var(Z_T|\Omega_\tau) &= h^2 (Q^b - Q^a)^2 var(F_T|\Omega_\tau) \\ 2cov(X_T, Z_T) &= 2h (Q^b - Q^a)^2 [cov(B_T, F_T|\Omega_\tau) + cvar(F_T|\Omega_\tau)] \end{aligned}$$

as  $E(Z_T|\Omega_\tau) = var(Y_T|\Omega_\tau) = cov(X_T, Y_T) = 2cov(Z_T, Y_T) = 0$ . Substituting these results into equation (9) and optimize with respect to  $h$  yields

$$h^* = - \left[ \frac{cov(B_T, F_T|\Omega_\tau)}{var(F_T|\Omega_\tau)} + c \right]$$

where  $h^*$  is the hedge ratio and  $h^*(Q^b - Q^a)$  the fraction being hedged by the market maker.

2. The exact simulation process goes as follows:

- (a) We start with the following parameter values:  $F_0 = 100, c = 0.95, \beta = 0.99, \xi_a = \xi_b = 50, X_{t_0} = 10, r = 0$  and  $\tau = 500$ . We also set  $\sigma_f = 1$  and let  $\sigma_b$  vary from 1 to  $20\sigma_f$  (in steps of one),  $\sigma_{buy} = 1$  and  $\sigma_{sell}$  varies from 1 to  $1.10\sigma_{buy}$  (in steps of 0.01).
- (b) Given these standard deviations, we simulate  $\varepsilon_{t,f}, \varepsilon_{t,b}, \varepsilon_{sell}$  and  $\varepsilon_{buy}$ .
- (c) We calculate the starting values  $B_0 = (1-c)100 = 5$  and  $P_0 = B_0 + cF_0 = 100$ . Using these starting values and the residuals simulated in step 2, we calculate  $F_t, B_t$  and  $P_t$  for  $t = 1, \dots, 100$ .
- (d) The quoted price is concentrated around the expected bond price at time  $T = 1000$  (we set  $E_\tau(P_T) = P_\tau$ ). The market maker chooses a bid-price  $E_0P_T - 0.5S$  and an ask-price  $E_0P_T + 0.5S$ . Given these bid and ask prices, we can calculate the incoming and outgoing



order flows at time  $t = 1, \dots, \tau - 1$  and hence the cash position  $Y_{\tau-1}$ . Because  $r = 0$ , we have  $Y_{\tau-1} = Y_T$ . Based on the simulated prices from  $t = \tau, \dots, 1000$ , we calculate the value of the inventory and margin account (i.e.  $Z_T$  and  $X_T$ ). From these 1000 observations, we can also calculate the expected wealth and the associated standard deviations. The utility function is maximized, by choosing the optimal spread  $S^*$ , subject to the constraints that the expected wealth and the spread are nonnegative. We also calculate the optimal spread  $S^*$  under  $\sigma_{sell} = 1.01, \dots, 1.10$  (in steps of 0.01).

(e) Step (b) to (d) is repeated for  $\sigma_b = 2, 3, \dots, 20$  (in steps of 1).

We replicate 200 times step (b) to (e) using different simulation seeds and the resulting spread is averaged out, i.e.  $\bar{S} = \frac{1}{200} \sum_{i=1}^{200} (S_i | \sigma_b, \sigma_{sell})$  is depicted in figure 3.

3 To see the relation between the spot and futures market, consider a cash-and-carry strategy. At time  $t$ , buy EUR 100.000 face value of delivery bond  $i$  at price  $P_{i,t}$  while simultaneously selling a futures contract with a price  $F_t$ . Finance this transaction using a repo agreement with interest rate  $R$  and hold this bond until delivery date  $T$ . The following cash flow arises for the short at delivery date:

- Receive the invoice price (equation 17);
- Buy this bond, repo out and receive the  $P_{i,t} + acc_{i,t}$ . At time  $T$ , the short has to reverse the repo and pays back <sup>23</sup>:

$$(P_{i,t} + acc_{i,0}) \left( 1 + R \frac{T-t}{365} \right) \quad (38)$$

- Under a repo, the coupon and its accrued interest belongs to the original holder rather than the repo trader. This means that at time  $T$ , the original holder receives

$$\sum_{i=1}^N \gamma_i \left( 1 + R \frac{T-t_i^*}{365} \right) \quad (39)$$

where  $t_i^*$  is the date in which coupon  $coupon_i$  is paid back to the original holder of the bond.

The net profit of this cash and carry strategy must be zero as it is (virtually) risk free for default free bonds. Hence, the relationship between the spot and the futures price is given by equation (40):

$$c_i F_t = (P_{i,t} + acc_{i,0}) \left( 1 + R \frac{T-t}{365} \right) - \sum_{i=1}^N \gamma_i \left( 1 + R \frac{T-t_i^*}{365} \right) - acc_{i,T} \quad (40)$$

This tells us that the futures price follows the price of the underlying deliverable bond, its repo rate and time until maturity. All the parameters needed to calculate the fair price of the futures contracts are

<sup>23</sup>In this example we assume an actual/365 basis.

known in advance<sup>24</sup>. The analysis shows that we can calculate the fair price of a futures contract given the CTD bond. It does not say anything about the stochastics of the fair future price because it does not provide a mechanism to analyze the change in the CTD. At any time, we can calculate which bond is the CTD as anyone who conducts this cash and carry strategy will choose a bond such that the net profit when delivering the bond against its short futures position is maximal.

$$\max c_i F_t + acc_{i,T} + \sum_{i=1}^N \gamma_i \left( 1 + R \frac{T - t_i^*}{365} \right) - (P_{i,t} + acc_{i,0}) \left( 1 + R \frac{T - t}{365} \right) \quad (41)$$

In order to find the cheapest to deliver bond, one can calculate the net profit as given by (41) for all the deliverable bonds. The bond that gives the highest net profit is the cheapest-to-deliver. Using arbitrage arguments, this net profit will be equal or below zero and the bond with the lowest fair futures, price is the CTD.

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<sup>24</sup>Running a cash and carry strategy also involves the paying of a repo rate while at the same time earning interest on the coupons. When the coupon rates are higher than the repo rates, we have a positive carry. As a result, future prices with a longer delivery date have a lower price as it cost less to conduct the strategy. On the other hand, if we have negative carry, the futures price for securities with a longer delivery date is higher.

## 6 Graphics

Figure 1: Yield differences of 10-year government bonds issued by various Eurozone. The Y-axis indicates the differences in basis points. The X-axis reflects data running from 3 November 2003 until 3 November 2004 giving a total of 263 daily observations. As we can see, the 10-year Dutch State loans are trading (on average) below its French equivalent. The Italian 10-year bond is trading flat against Greece but at a yield pick-up against Portugal.

Figure 2: Snapshot of prices from running benchmark bonds as observed on Tradeweb in November 2003. The differences in yields can also be observed in here.

Figure 3: Simulation of 200 times 1000 buy and sell orders. The details of this procedure can be found in the appendix (point 2). The volatility ratio is given by  $\phi = \frac{\sigma_{sell}}{\sigma_{buy}}$ . We let basis risk run from  $\sigma_F$  to  $20 \times \sigma_F$ .

Figure 4: Daily turnover on the Eurex-system in terms of contracts per day, the number of trades, average trading size and volume weighted price for the June 2001 contract as observed in our dataset. The number of contracts and the number of trades per day picks up considerably around day 40 (March 2001) when the June 2001 became the front contract. At the same time, the number of contracts traded per day drops.

Figure 5: Daily turnover on the Eurex-system in terms of contracts per day, the number of trades, average trading size and volume weighted price for the September 2001 contract as observed in our dataset. The number of contracts and the number of trades per day picks up considerably around day 110 (June 2001) when the September 2001 became the front contract. At the same time, the number of contracts traded per day drops.

Figure 6: Average absolute sample mean of the Bund Future. Figure shows a broad U-shape and is closely linked to the cycle of market activity. Volatility is gradually increasing until 10.00 CET (interval 24) after which volatility drops with a low around the lunch hours. Market volatility is relative quiet but starts picking up around 14.30 CET (interval 74) in which market makers are preparing themselves for the opening of the US markets.

Figure 7: Plots of the unobservable state variables for the 2011 bonds calculated using the Kalman filter. Here, FGBL refers to the efficient futures price.

Figure 8: Plot of the residuals from the Kalman filter process based on a Choleski decomposition of  $F_t^{-1}$ . The figures show that the residuals are stationary.

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