



Average Interest Cost – A Novel Cost and Risk Measure and its Application in Public Debt Management^a

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1 Abstract

We present the novel cost and risk measure “Average Interest Cost” (AIC) as a highly appropriate cost metric for the purpose of managing public debt. The AIC is the average interest cost over a long time horizon. It incorporates the historic interest expenses, the difference between market and par value of the portfolio and expected future cost for refinancing for a long time horizon. We highlight five key requirements for a cost measure in public debt management (i.e. capturing every cashflow, realistic perspective on long-lasting public debt, rising cost with rising interest rates, risk as standard deviation of cost, and longer portfolios implying lower risk). We show that, unlike other used cost measures, the AIC fulfills all of them. Furthermore, the AIC allows intuitive understanding and is easy to use in practice. In addition to the specific parametrization used in the management of the German federal debt portfolio, we present possible modifications for other debt managers. We conclude with fictional analyses based on use cases from the German Finance Agency, including historic performance analyses, deterministic future scenario analyses, and strategical portfolio decision.

Keywords: public debt management, government bond market, interest cost, market risk, portfolio decision, issuance strategy

JEL Classification: H63, H68, G11, G12, G17

2 Introduction – Cost in Public Debt Management

One of the main challenges of public debt management is the choice of a specific portfolio composition. There are numerous demands that typically every considerable debt portfolio has to fulfill like the financing needs, the cash management, the liquidity of the market, regulations and others. These “exogenous” aspects limit the choice of an issuance strategy, yet there remains substantial space for decisions between different strategies that all fulfill these exogenous demands. The decision between these issuance strategies is one of the key tasks of public debt management. It is widely accepted that this

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decision should be based on minimizing cost while limiting associated risk, see for instance (Lewis & Viñals, 2014), (Jonasson & Papaioannou, 2018), (Jonasson, et al., 2019) and (Andabaka Badurina & Svaljek, 2012).

The main contribution of this paper is the introduction of a novel cost measure for public debt, the Average Interest Cost, and its corresponding risk measure. This cost measure builds a bridge between the two most common approaches to determining the cost associated with public debt, adapting the advantages of both approaches. The paper provides a detailed derivation of the Average Interest Cost and presents different use cases from the perspective of a debt manager.

In this chapter we outline the challenge of defining a suitable cost measure to fulfill key requirements specific for public debt management. Chapter 3 follows with the motivation and definition of the proposed novel cost measure with a minimum of technicalities. Chapter 4 provides a more in-depth view on the cost and associated risk measure and we show that the proposed cost measure actually fulfills all key requirements introduced in Chapter 2. In Chapter 5 we present some illustrative use cases from a practitioner's view, followed by concluding remarks in Chapter 6.

2.1 Cost Measures for Management of Public Debt

While it seems clear that cost and risk should be the leading measures for decisions regarding the structure of the debt portfolio, it is not generally defined how to measure cost and risk. Here we focus only on cost due to interest expenses, since they directly impact the budget. For a sizable debt portfolio these cost usually outweigh other cost components, like transaction cost. Additionally, we focus on the associated market risk that needs to be restricted for sustainable debt management. But even with this focus it is not obvious how to define these cost and risk measures.

In the literature there are two different school of thoughts to approach cost for public debt management:

a) *Budget Approach*

The first, more common approach is mostly used by the practitioners in public debt management organizations. The used cost measures add up the interest expenses and therefore the budgetary burden (see e.g. (European Commission & Eurostat, 2013) for accounting standards) over multiple years. For a historic timespan this cost measure can be calculated straightforward. But it is also possible to determine future interest expenses, by incorporating not only those associated by the current portfolio but also considering those generated by rolling over maturing debt before the end of the specified period. For this cost measure looking into the future it is necessary to define and implement a strategy for the rollover debt and a development of the yields. For more details see (Risbjerg & Holmlund, 2006) for a comprehensive overview to a general framework, or (Bernaschi, et al., 2019), (Yavuz & Gölbasi, 2019), (Bergström, et al., 2002), (Bolder & Deeley, 2011), (Larson & Lessard, 2011) and (Bitner, et al., 2024).

b) *Mark-to-Market Approach*

The second approach is a mark-to-market view and is based on the market value of the debt portfolio. The change in market value can be interpreted as cost, similar to the return of an asset portfolio. Due to the close connection to common practices in asset portfolio management, there are suggestions to apply this measure within the domain of public debt management (see e.g. (Hall, et al., 1997), (Bernaschi, et al., 2009), (Berndt & Yeltekin, 2015), (Ellison & Scott, 2020)).

Both of these approaches have its advantages and drawbacks as will become clear in the following sections. Additionally, in (Blommestein & Hubig, 2012) it is argued that following these different approaches might lead to different portfolio strategies. A suitable cost and risk measure for public debt management combines all advantages of the above approaches with as few drawbacks as possible.

In this paper we focus on a sustainable long-term perspective on managing public debt. We assume in the following that the issuer has a well-established access to the capital market and that the debt is expected to last much longer than the average maturity of the current debt portfolio. Additionally, we expect that



the issued instruments are redeemed at the end of lifetime and are not bought back in a significant amount, analogous to a buy-and-hold-strategy.¹

2.2 Key Requirements for Decision Making in Public Debt Management

In the following we introduce five key requirements of a cost and risk measure that are important for debt management and the strategy choice in particular. We also evaluate whether the two standard approaches mentioned before actually incorporate these requirements.

2.2.1 Capturing Every Cashflow

The requirement of capturing every cashflow regards to the fact that all cashflows of all issued instruments have to be taken into account for the cost measure. Otherwise it might be possible that unintended window dressing emerges, and a strategy might seem to generate less cost while in reality it only shifts cashflows into the future. Incorporating all cashflows ensures a complete economic view on the debt portfolio.

The mark-to-market approach actually fulfills this requirement by design. All cashflows of the portfolio are discounted to the present value, no matter how far in the future they might be, or whether the cashflows are already fixed or not.

The budget approach, instead, only includes those cashflows that fall within the considered timespan. However, there is still a debt portfolio with outstanding debt at the end of this timespan. Since this portfolio is typically not priced at par at the end of the timespan, a relevant component is not considered in this approach. This effect can be neglected for infinitely long timespans, however e.g. for a future view, most simulations do not consider more than 10 years. In (Dottori & Manna, 2016) the authors examine a model of public debt management and show analytically that the choice of the optimal portfolio highly depends on the considered simulation horizon. This effect is deeply linked to the fact that in the budget approach all portfolio effects after the end of the considered timespan are neglected. This is also already pointed out by (Bernaschi, et al., 2009): “In fact, portfolio strategies cannot be compared in terms of interest costs unless the time horizon extends up to the redemption date of the longest maturity bond issued over the simulation period.” These authors instead favor a return-based asset-liability view.

2.2.2 Realistic Perspective on Long-Lasting Public Debt

The requirement of a realistic perspective on long-lasting public debt reflects the general observation that public debt prevails much longer than the average time to maturity of the outstanding debt. Most of the debt portfolio is expected to be rolled over by issuing new bonds after the redemption of the existing ones.

This aspect is incorporated in the budget approach for the considered timespan. In a historic view this is obvious, since the budgeted approach always pertains to the actual debt portfolio. For a future looking view, this requirement is explicitly taken care of by specifying an issuance strategy to accommodate the debt requirements.

The mark-to-market approach however, does not consider the fact of a long outstanding debt at all. This approach implicitly assumes that a quick redemption of the portfolio is possible. While this is appropriate for many asset managers, it is typically not for public debt management.

¹ Even if not all assumptions hold in a perfect manner, the suggested cost measure might still offer significant insights for the debt manager.



2.2.3 Rising Cost with Rising Interest Rates

For public debt it is intuitively obvious that the considered cost of the debt service has to increase, when interest rates rise, and vice versa. Even if the complete debt is financed via fixed bonds, higher interest rates imply higher coupons for the debt refinanced in the future.

This is captured in the budget approach, since rising interest rates results in higher coupons for the newly issued bonds and therefore to a rise of the cost. The mark-to-market approach however shows exactly the opposite behavior. If interest rates rise the present value of the outstanding debt decreases and vice versa². This view might be appropriate if the debt can actually be redeemed at any time. But typically, as already pointed out, public debt has to be refinanced at its maturity and therefore higher cost due to the higher interest rates are almost inevitable.

2.2.4 Risk as Standard Deviation of Cost

As mentioned before, sustainable public debt management should not only focus on cost reduction but also limit or control the risk associated with the debt portfolio. The fluctuation width of the chosen cost measure has to lead to a meaningful measure for market risk. Fluctuations could result from either time series with actual historic movements or from the distribution of different stochastic or deterministic scenarios. The uncertainty could be measured by variance, percentiles or the standard deviation. The close link between cost and risk measure makes it easy to weigh up cost versus risk intuitively and facilitates the comparison of different portfolios in a consistent manner. In the end, even portfolio optimizations based on the portfolio theory by Markowitz (Markowitz, 1952) can be performed.

For an appropriate cost measure for public debt management it has to be possible to define a meaningful risk measure in this context. For both approaches described in 2.1 this is straightforward.

2.2.5 Longer Portfolios Imply Lower Risk

Having a sustainable long-time view, it seems clear for public debt management that longer portfolios imply lower risk than a shorter portfolio: A longer portfolio (higher average time to fixing) is less sensitive to movements of the yield curve, since a higher percentage of the future coupons are known and fixed and a smaller percentage has to be re-fixed, or refinanced. Therefore, the cost of a longer portfolio fluctuates less than the cost of a shorter portfolio and in turn implies lower (refinancing) risk.

This behavior is covered by the budget approach by construction. The mark-to-market approach instead shows again the opposite effects. A longer bond has more and later fixed cashflows than a shorter one. The present values of all these cashflows are sensitive to the movement of the yield curve. Therefore, the present value of a longer portfolio fluctuates more with interest rate fluctuations than that of a shorter portfolio and therefore shows larger risk (see also (Hall, et al., 1997)).

3 Presenting Average Interest Cost

The Average Interest Cost presented in this chapter combines the best of the two approaches in 2.1 to accommodate all key requirements listed in 2.2 and provides a complete and appropriate cost measure.

Throughout this paper, we use a sign convention with positive debt, positive present value, and positive interest expenses in the case of positive interest rates. However, the arguments presented are invariant under a thorough change of sign convention.

² We presume positive present value, for more details see chapter 3.



3.1 Defining Average Interest Cost

The main idea of the Average Interest Cost is to add up historic and all expected future interest expenses over a long time and to average the expenses over the considered timespan. This leads to a highly condensed metric that indicates the expected average interest expenses in this time interval.

The historic interest cost can be easily measured following the budget approach, but determining the expected future interest cost is the main challenge. The expected future interest cost can be determined via an additional simulation over a long time horizon under forward development of yields using the budget approach again. However, inspired by the mark-to-market approach, these future interest cost can be completed by considering the agio of a hypothetical rebuy of the portfolio at the end of this considered period. Since this method requires the specification of a refinancing strategy, the AIC uses an even simpler approach that coincides with that method under the forward development: The expected future interest cost is calculated by a hypothetical rebuy of the debt portfolio at current time t and a refinancing of the debt up to a future time point at current market conditions. In the following this idea will be described in more detail.

The AIC can be split into three parts defined as follows.

- **Historic Interest Expenses:** The historic interest expenses $IE(t_0, t)$ can be seen as the actual accumulated interest expenses from the starting point t_0 to the current point in time t . The accumulation of past interest expenses is necessary for a fair comparison of two different strategies, since two strategies can differ significantly in the distribution of cashflows over time.³ This has to be considered when choosing the starting point t_0 , so that no strategies gain an artificial advantage. This reflects to the requirement of capturing every (relevant) cashflow for the historic interest expenses. Apart from that, t_0 can be chosen pragmatically. It is usually useful to start at a significant point, e.g. a general change in strategy, a new market era or something similar.
- **Market Value Compensation (MVC):** The second part of the AIC, the so-called “Market Value Compensation” (MVC) is defined as

$$MVC(t) = PV(t) - D(t),$$

where $PV(t)$ is the present value and $D(t)$ the debt of the current portfolio at time t . This part can be interpreted as the cost at time t of a hypothetical rebuy of the complete debt portfolio at the current price, combined with the refinancing of the debt at par value. If the debt portfolio already only consists of par bonds regarding the market conditions at time t , the present value is equal to the debt generated by the portfolio and therefore the Market Value Compensation is zero. All deviations of the portfolio from current market conditions are directly reflected in this term.

- **Refinancing Cost:** The third part, the financing cost over the future horizon ΔH , or “Refinancing Cost”

$$RefiCost(t) = D(t) \cdot \Delta H \cdot \bar{z}(t, t + \Delta H),$$

indicates the future interest expenses of the above-mentioned refinancing at par value. It can be interpreted as a hypothetical single bond for the complete debt over a given AIC-Horizon ΔH with the refinancing yield $\bar{z}(t, t + \Delta H)$, for details see 3.3.

The sum of these parts defines the “Total Interest Cost” (TIC)

$$TIC(t) = IE(t_0, t) + PV(t) - D(t) + D(t) \cdot \Delta H \cdot \bar{z}(t, t + \Delta H),$$

and averaging over the considered timespan yields the Average Interest Cost (AIC)

³ In a simulation-based view into the future, these historic interest expenses correspond to the accumulated interest expenses in the simulation timespan, as depicted in the use case in 5.2.



$$\begin{aligned}
 AIC(t) &= \frac{TIC(t)}{t - t_0 + \Delta H} \\
 &= \frac{1}{t - t_0 + \Delta H} (IE(t_0, t) + PV(t) - D(t) + D(t) \cdot \Delta H \cdot \bar{z}(t, t + \Delta H)).
 \end{aligned}$$

The Average Interest Cost is therefore the mean annual cost generated by the debt portfolio considering the past period since t_0 and the future period up until $t + \Delta H$.

The inspiration from the two approaches in 2.1 is getting clearer by a detailed look at the AIC. In the Market Value Compensation, the close connection to a mark-to-market approach becomes obvious. The AIC is closely connected to the market value of the current portfolio and the Market Value Compensation moves according to market movements. In contrast to the mark-to-market approach, the hypothetical rebuy is appropriate here, because it coincides with the refinancing. The inclusion of the historic interest expenses and especially the Refinancing Cost in contrast illustrates the close link to the budget approach. However, AIC employs a universal Refinancing Cost, independent of the current portfolio structure with an implied development of yields according to the forward path. This provides a simple comparison of portfolios, while being unaffected regarding shifts of cashflows.

3.2 A Coupon Based View on AIC

There is a second interpretation of the AIC or TIC, respectively. Under some approximations it can be shown that the AIC corresponds to the accumulated interest expenses including the historic interest expenses and also the future interest expenses up to the defined horizon. For illustration purposes consider a single bond. For each bond in the debt portfolio it is possible to determine the expected future interest cost by adding two parts. The first part is the already known future interest cost due to the fixed coupon of this existing bond. The second part, the unknown future interest cost, accounts for refinancing the existing bond after its maturity up to the end of the horizon $t_H = t + \Delta H$, see Figure 1 for a depiction. The unknown future refinancing cost can be determined by the forward implied interest cost of a bond lasting from maturity of the existing bond to the end of the horizon.

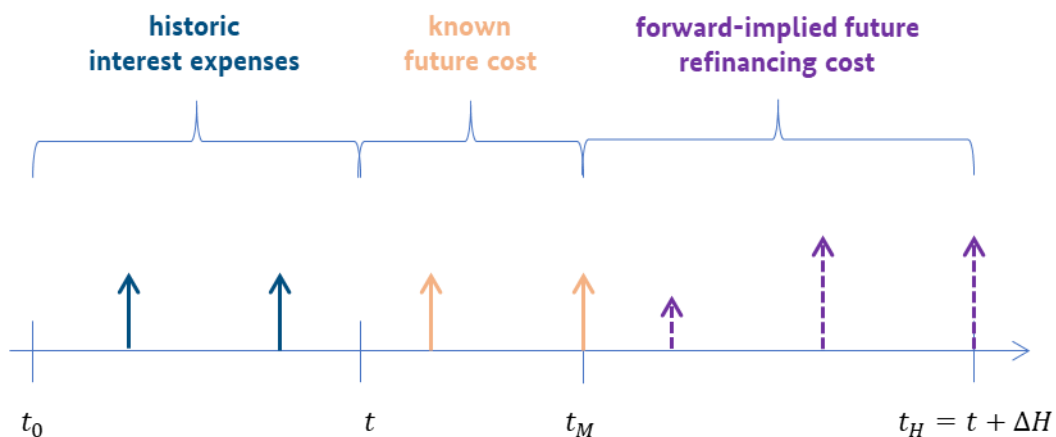


Figure 1: Schematic depiction for a coupon-based view on AIC.

By applying this view to each outstanding bond of the current portfolio, it is ensured that the complete debt is refinanced exactly and only up to the end of the AIC-Horizon⁴. With some approximation (and apart from discounting) the sum of these already known interest cost and the unknown interest cost

⁴ In this view, bonds maturing after the end of the AIC-Horizon have to be bought back at the end of the AIC-Horizon to ensure, that the debt is only financed up to that point. The corresponding agio has to be considered as well for a match with the actual formula of AIC.



determined under current market conditions, correspond to the sum of the above defined Market Value Compensation and Refinancing Cost. An approximate derivation can be found in Appendix A.

3.3 The Horizon, The Refinancing Yield and Their Modification

To guarantee that the AIC actually fulfills the key requirements mentioned in 2.2, it is essential to specify the AIC-Horizon ΔH in combination with the refinancing yield $\bar{z}(t, t + \Delta H)$ thoroughly. This becomes clear, by stressing the image of rebuying the debt portfolio and issuing a new single bond at then current market prices to refinance the debt,

The considered horizon ΔH has to be long compared to the average time to maturity of the longest realistic debt portfolio, but does not need to exceed the time to maturity of the longest instrument itself. It has to be longer than the typical average lifetime of a portfolio because otherwise the key requirement of longer portfolios providing lower risk will not hold anymore. For details see chapter 4. On the other hand, if the horizon is too long, the AIC is dominated by the refinancing term, which is the same for all portfolios. This would diminish deviations of the AIC generated by different portfolio strategies. As a rule of thumb, a horizon which is half as long as the longest instrument ensures this property, as it corresponds to a stable strategy issuing each the same amount of only new bonds with this longest possible maturity each year.

It is important to note that the implied refinancing strategy, in our modelling a single bond, is technically a “risk-free” strategy. Since there is no risk-free strategy in the real world of public debt management, the implied refinancing strategy has to be modeled to be not achievable by the debt managers via realistic issuance strategies. This can lead to different refinancing strategies for different debt managers, depending on the debt portfolio and specific views of the debt manager.

In the management of the federal debt the German Finance Agency uses a time horizon of the AIC of $\Delta H = 15$ years, since the longest issued German bond has a time to maturity of 30 years. We consider this a suitable trade-off between long enough and as short as possible.

The refinancing yield $\bar{z}(t, t + \Delta H)$ is the second important part of the Refinancing Cost in the AIC. This refinancing yield has to have close connections to the current yield curve at time t and has to reflect a suitable yield for the time to maturity of ΔH . The most straightforward approach is the zero yield i.e. $\bar{z}(t, t + \Delta H) = z(t, t + \Delta H)$. However, other definitions and modifications are possible, see Appendix B for details.

3.4 Incorporating Other Instruments

The general idea of the AIC, rebuying the portfolio and refinancing the debt via a “risk-free” strategy, is also compatible with inflation linked bonds, foreign currency bonds or swaps and other derivatives. All properties of these instruments are fully incorporated in the present value and therefore contribute completely and directly to the AIC.

Instruments that generate debt, like inflation linked bonds and foreign currency bonds, also influence the AIC via the considered Refinancing Cost. Swaps and other derivatives, however do not contribute to the debt and therefore have no impact on the refinancing part of the AIC.

Assuming a (sub)portfolio consisting only of swaps, there is no debt and therefore no Refinancing Cost. In this case the AIC matches the total return in close relationship to an asset manager’s view and the mark-to-market approach in 2.1. Here, the lack of Refinancing Cost is appropriate, since such a derivative portfolio could actually be terminated on a short time horizon without implying any significant refinancing needs in contrast to the actual debt portfolio.



4 Properties of Average Interest Cost and its Risk

Based on the definitions of the AIC in 3.1 we show in this chapter that the AIC actually incorporates all key requirements from 2.2. With the help of some detailed views and definitions it becomes evident that the AIC is an appropriate cost measure for public debt management.

4.1 Incorporating Every Cashflow

From the definition in 3.1, it becomes clear that the AIC or TIC actually captures every cashflow in the sense described in 2.2.1. Using the present value of the complete portfolio for the Market Value Compensation and historic interest expenses, all cashflows of the portfolio are always considered in this approach no matter which timespan is considered. This property is directly inherited from the present value. Additionally, the Refinancing Cost is by design a complete closure of the expected future cost: The refinancing of the debt via a single bond with maturity at $t + \Delta H$ is the same for all portfolios, independent of the specific portfolio structure. Therefore, using the AIC every portfolio is treated equally, and thereby ensuring a fair evaluation at the end of the AIC-Horizon $t + \Delta H$.

4.2 Reflecting a Realistic View on Long-Lasting Public Debt

The key requirement described in 2.2.2 regarding the general observation of long-lasting public debt is ingrained in the definition of the AIC. The Refinancing Cost covers the cost associated with the debt lasting up to the end of the AIC-Horizon $t + \Delta H$. This point in time can differ among debt managers. However, the AIC-Horizon is expected to be significantly longer than the typical average time to maturity of the considered debt portfolio. The Refinancing Cost can also be easily adjusted, e.g. to accommodate an anticipated path of debt reduction or debt increase. Parametrizing the Refinancing Cost in such a way that it reflects a realistic view on the specific debt portfolio is straightforward following the guiding principles presented in 3.3.

4.3 Cost Characteristics: Rising Cost with Rising Interest Rates

The AIC is highly correlated with the movement of interest rates. For a better understanding of this, it is useful to define a new key figure, the $TIC01(t)$, that indicates the change of the Total Interest Cost due to a yield curve shift of 1 bp. It is determined via the derivative of the Total Interest Cost with respect to parallel shifts of the yield curve:

$$\begin{aligned} TIC01(t) &= 10^{-4} \cdot \frac{d}{dz} TIC(t) \\ &= 10^{-4} \cdot \frac{d}{dz} (PV(t) + D(t) \cdot \Delta H \cdot \bar{z}(t, t + \Delta H)) \\ &= PV01(t) + 10^{-4} \cdot D(t) \cdot \Delta H. \end{aligned}$$

The $TIC01(t)$ shows a close relationship with the $PV01(t)$, however it has the opposite sign. From the issuer's perspective the $PV01(t)$ of a debt portfolio is negative and therefore rising interest rates lead to a smaller present value. Due to the second term in the $TIC01(t)$, the sign switches, if ΔH is large enough. The $TIC01$ becomes positive, and therefore the TIC (and with that the AIC) actually increases with rising interest rates.

With a rough approximation of $PV(t) \approx D(t)$ and the definition of the duration as

$$Duration(t) = -\frac{PV01(t)}{PV(t)},$$

the $TIC01$ can become even more intuitive:

$$TIC01(t) \approx 10^{-4} \cdot D(t) \cdot (\Delta H - Duration(t)).$$



The $TIC01$ is positive when the considered AIC-Horizon is longer than the typical duration of the portfolio. The arguments in 3.3 for a suitable length of the horizon are actually based on this view. By choosing a long enough AIC-Horizon, the AIC shows the suitable cost characteristic.

It is also possible to define the AIC01 based on the TIC01

$$AIC01(t) = \frac{TIC01(t)}{t - t_0 + \Delta H}.$$

Due to the averaging over the complete timespan and the corresponding damping effect this key figure has to be used with caution when examining the development over time. For evaluating the development of the portfolio composition over time, the TIC01 provides a more intuitive interpretation.

4.4 AIC-Risk Based on Historic Increments: Risk as Standard Deviation of Cost

The interpretation of the historic AIC-Risk as the fluctuation of the AIC, is straightforward. We define historic increments of the TIC from dates t_1 to t_2 as

$$\delta TIC(t_1, t_2) = TIC(t_2) - TIC(t_1) - \delta D(t_1, t_2) \cdot \Delta H \cdot \bar{z}(t_2, t_2 + \Delta H),$$

which indicates the change in TIC from one timepoint to the next one. The last term is a correction term with

$$\delta D(t_1, t_2) = D(t_2) - D(t_1),$$

that offsets possible fluctuations in the TIC due to changes of the amount of debt. These changes in debt are no source of market risk and should therefore be excluded.

Based on these historic increments, it is possible to use statistical methods to derive all kinds of variance measures. In the German debt management, we use standard deviation on the N daily increments of the last year,

$$TIC_{Risk}(t) = \sqrt{\frac{1}{N-1} \sum_{\tau=t-1\text{year}}^t (\delta TIC(\tau-1\text{day}, \tau) - \overline{\delta TIC}(t))^2},$$

with the corresponding mean increment

$$\overline{\delta TIC}(t) = \frac{1}{N} \sum_{\tau=t-1\text{year}}^t \delta TIC(\tau-1\text{day}, \tau).$$

Other statistical measures and other timespans are of course also possible.

For the calculation of the AIC-Risk, the TIC-Risk has to be normalized by the considered timespan. This can be done in two different ways, depending on preferences. First, normalizing by the same timespan as the AIC itself, using

$$AIC_{Risk}^{long}(t) = \frac{TIC_{Risk}(t)}{t - t_0 + \Delta H}.$$

This incorporates the fact that the AIC will become more and more stable, the further t_0 lies in the past: While only the Market Value Compensation and the Refinancing Cost actually create risk, with a longer history the historic interest expenses start dominating the AIC. Therefore, the $AIC_{Risk}^{long}(t)$ will become lower due to normalization even if neither the portfolio nor the market volatility changes.

The other approach is normalizing only by the future timespan, i.e. the AIC-Horizon ΔH . This slightly decouples the AIC-Risk from the AIC, but has a stronger connection to the current portfolio and its composition. Also, in a time series of the AIC-Risk it is easier to compare the risk at two different timepoints.



In German debt Management the latter perspective is the preferred one and the AIC-Risk is generally given by

$$AIC_{Risk}(t) = \frac{1}{\Delta H} \cdot TIC_{Risk}(t).$$

4.5 Risk Characteristics: Longer Portfolios Imply Less AIC-Risk

The risk characteristics of the AIC can also be examined by using the aforementioned $AIC01(t)$ and $TIC01(t)$. By using the approximation of

$$TIC01(t) \approx 10^{-4} \cdot D(t) \cdot (\Delta H - Duration(t)),$$

it becomes clear that the sensitivity of the TIC decreases when ΔH and the duration get closer. With a suitable choice of ΔH this means: The longer the duration of the portfolio, the less the AIC moves due to movements in the yield curve. This is exactly the intuitive behavior of longer portfolios implying less risk than shorter portfolios. Here also the interpretation of a “risk-free” refinancing strategy becomes clear: When the debt portfolio matches the refinancing strategy, the difference of ΔH and duration becomes zero, and the TIC is independent of shifts in the yield curve and therefore risk-free.⁵

Based on this interpretation, it might also be useful to define the TIC-Duration as

$$\begin{aligned} Duration_{TIC}(t) &= \frac{1}{D(t)} \cdot \frac{d}{dz} TIC(t) \\ &= \Delta H - \frac{PV(t)}{D(t)} \cdot Duration(t) \\ &\approx \Delta H - Duration(t). \end{aligned}$$

This TIC-Duration can be used in a similar way the usual duration is used in asset management.⁶ The larger the TIC-Duration (i.e. a small duration and usually a small average time to maturity), the more sensitive the portfolio is to changes in the yield curve and vice versa.

5 Comparison of Financing Strategies with AIC in Different Use Cases

Comparing different debt portfolios or strategies is essential for sound decision making in public debt management. In this chapter three analyses based on the Average Interest Cost are presented that are inspired by the use cases of the AIC in the German Finance Agency.

5.1 Historic Performance Evaluation

The evaluation of historic performance can provide significant insights into the strategic decisions made in the past and its consequences on the portfolio properties. In Figure 2 you can see the historic development of the AIC with starting date $t_0 =$ January 1st 2013 and an AIC-Horizon of $\Delta H = 15$ years for two different fictional portfolios. The development is based on the historic development of the German yield curve. The two portfolios, both financing a debt of 1 trillion €, are synthetic portfolios only constructed for illustration and are not realistic options. They can be characterized by their issuance strategy of issuing only either 2Y-Bonds or 20Y-Bonds, respectively. A detailed description of the construction of these portfolios including the development of the accumulated interest cost and the present value can be found in Appendix C. The two different issuing strategies, focusing either on short or long instruments, lead to a significant difference between the two strategies in terms of average time to

⁵ Here we neglect for simplicity the sensitivities to other movements of the yield curve, but the argument still holds: If the debt portfolio matches the refinancing portfolio exactly, the TIC will not change due to market developments.

⁶ For shortcomings of the standard definition of duration for public debt management, see (Jonasson & Papaioannou, 2018).



maturity and duration as well. However, due to construction, both strategies start at t_0 with the same PV and the same historic interest expenses and therefore with the same $AIC(t_0)$.

Looking at Figure 2, the high correlation of the AIC and the refinancing yield $\bar{z}(t)$ becomes apparent: A rising interest yield leads to rising AIC, showing one of the key requirements of a cost measure for public debt management. Additionally, it can be seen that the strategy with only 2Y-Bonds implies a shorter portfolio and therefore is more sensitive to the movement of the yield curve. The rising yields in 2012 or 2022 lead to larger increases of the AIC for the 2Y-Strategy than for the 20Y-Strategy. For the decline in interest rates in 2013 to 2020 the shorter strategy provides a larger decline of the AIC than the longer one. This emerges since the 2Y-Strategy has a higher percentage of the debt to be refinanced each year and therefore a stronger connection to the yield movements.

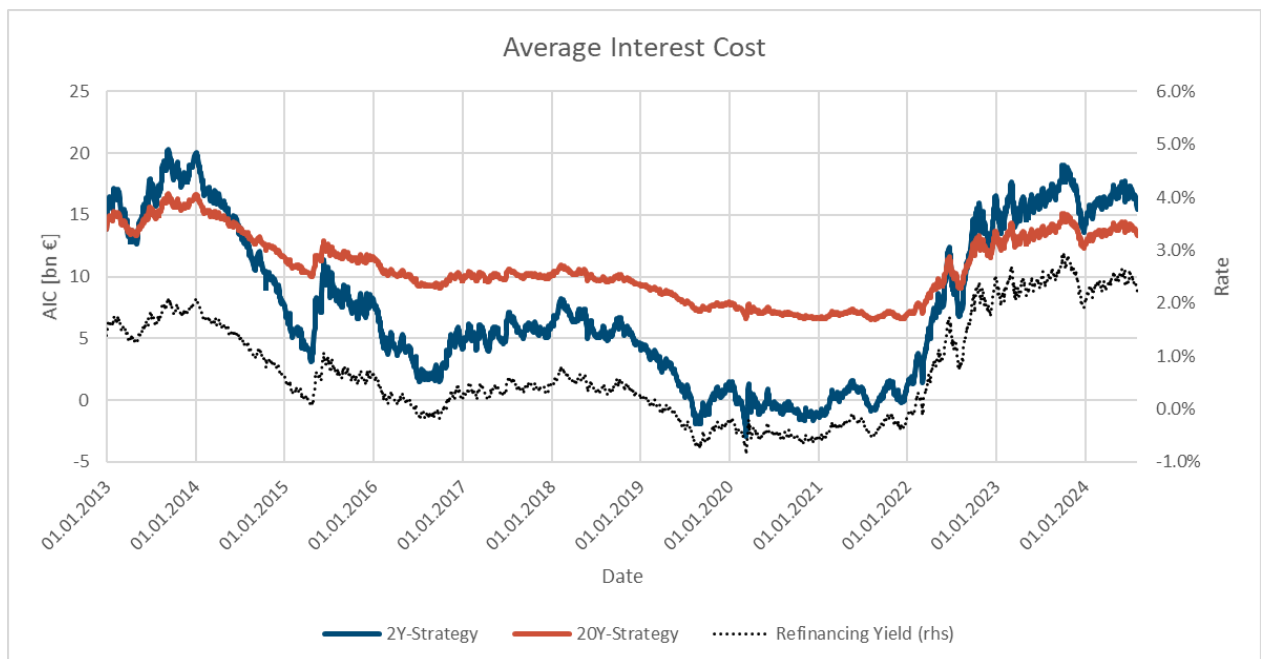


Figure 2: Development of the Average Interest Cost for two different strategies and the refinancing yield $\bar{z}(t)$ based on the yields of German Federal Bonds.

This observation also leads to the key requirement for public debt management regarding appropriate risk behavior. From the development of the AIC we can calculate the associated risk as the standard deviation of the daily TIC increments as described in 4.4, which is shown in Figure 3. The shorter portfolio shows a significant higher AIC-Risk due to the higher sensitivity to yield movements. Additionally, in times of larger volatility of the refinancing yield, the AIC-Risk increases for both strategies.

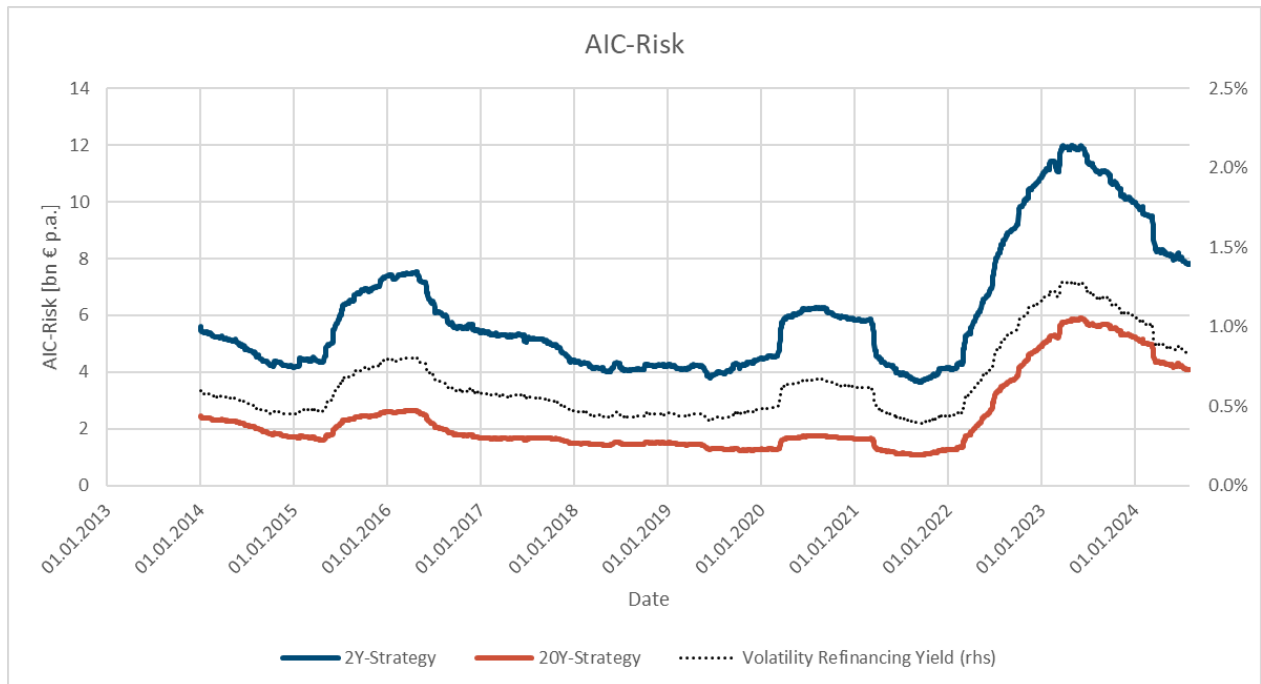


Figure 3: Development of the AIC-Risk for two different strategies as defined in 4.4 corresponding to the AIC shown in Figure 2 and the volatility of the refinancing yield.

The actual use cases for this historical analysis in the German Finance Agency are twofold. The historical development of the AIC and the AIC-Risk of the German debt portfolio is used for internal and external reporting. Reporting this cost measure provides a more structural view on the expected interest cost in the current market environment. While the actually paid interest expenses are important from a budget point of view, the structural long-term view of the AIC can provide a deeper understanding especially in times with rapidly changing yields, due to its direct response. The second use case for such a historical analysis is a complete and economic fair comparison of the actual portfolio against an alternative benchmark. Strategic decisions based on expectations of the development of the yield curve, say at the beginning of the year, can then be evaluated based on the actual realization of the yield development at the end of the year.

5.2 Future Scenario Analysis

In contrast to the historic evaluation in the previous section, decisions regarding a portfolio composition are made by looking into the future. This core task for public debt management is probably the most important use case for a cost measure in public debt management. Specifying different issuance strategies for the upcoming years and calculating cost and risk under different yield scenarios makes it possible to weigh one strategy against the other and make well-founded decisions. Using AIC for this future strategy evaluation is comparable to the established approach in public debt management already mentioned in the description of the budget approach in 2.1. In fact, the analysis based on AIC matches that approach, with only one major difference: The AIC also include the locked-in portfolio at the end of the simulation horizon and solving therefore the issue of neglecting future cashflows.

For this analysis we want to compare two different strategies and their development over time under different deterministic yield developments. Both strategies start at time $t_0 = \text{January 1st 2024}$ with the same portfolio and always finance a debt of one trillion €. They differ in the future issuance strategy by using only 2Y-Bonds, or only 20Y-Bonds, respectively, to rollover the debt. Over time this leads to different portfolios which also react differently to the market development. For details see Appendix C.2.

Figure 4 shows three simple deterministic scenarios for the development of the refinancing yield from January 1st 2024 to January 1st 2034. All scenarios start at a yield of 2.0% and include a linear yield shift



over the next 10 years. The first scenario for the refinancing yield is called the “expected path” and reflects the expectation of a fictive debt manager making the decision. This path corresponds to a moderate decline in yields of 100 bp over 10 years. The two other scenarios can be seen as stress scenarios corresponding to a 400 bp shift of the refinancing yield over 10 years upwards or downwards respectively.

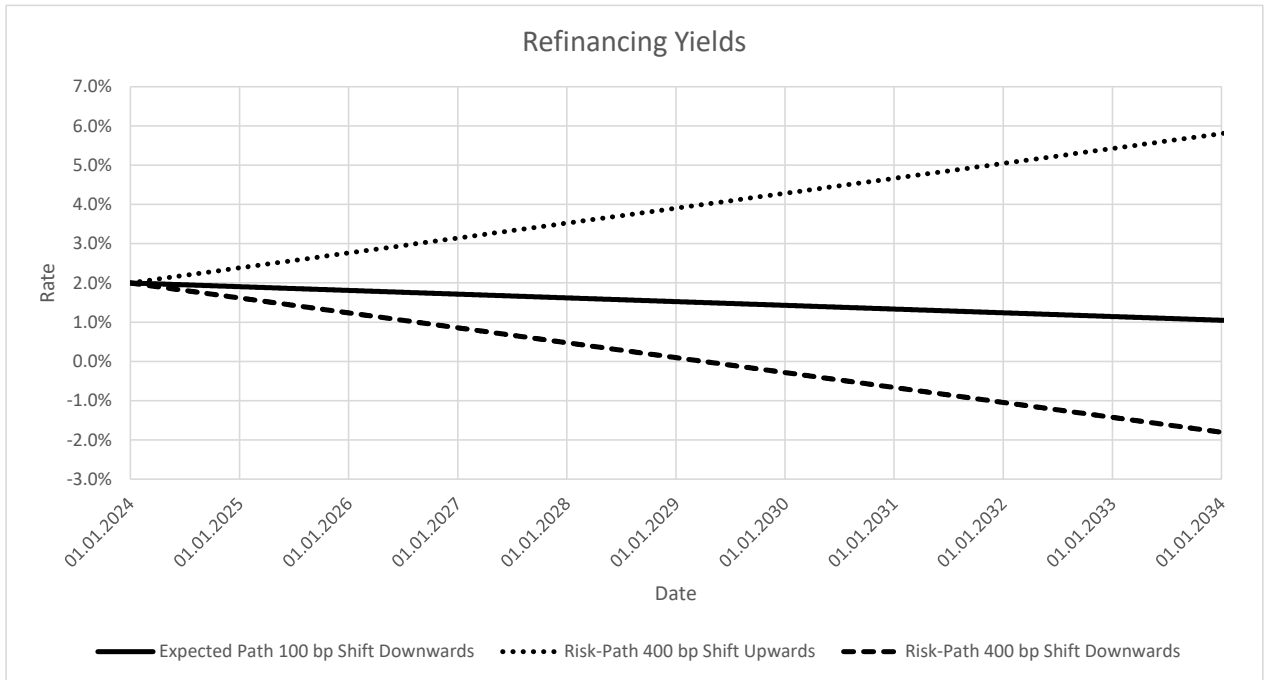


Figure 4: Different future deterministic paths of the refinancing yield $\bar{z}(t)$ corresponding to a linear shift over ten years of -100 bp, -400 bp and +400 bp, respectively.

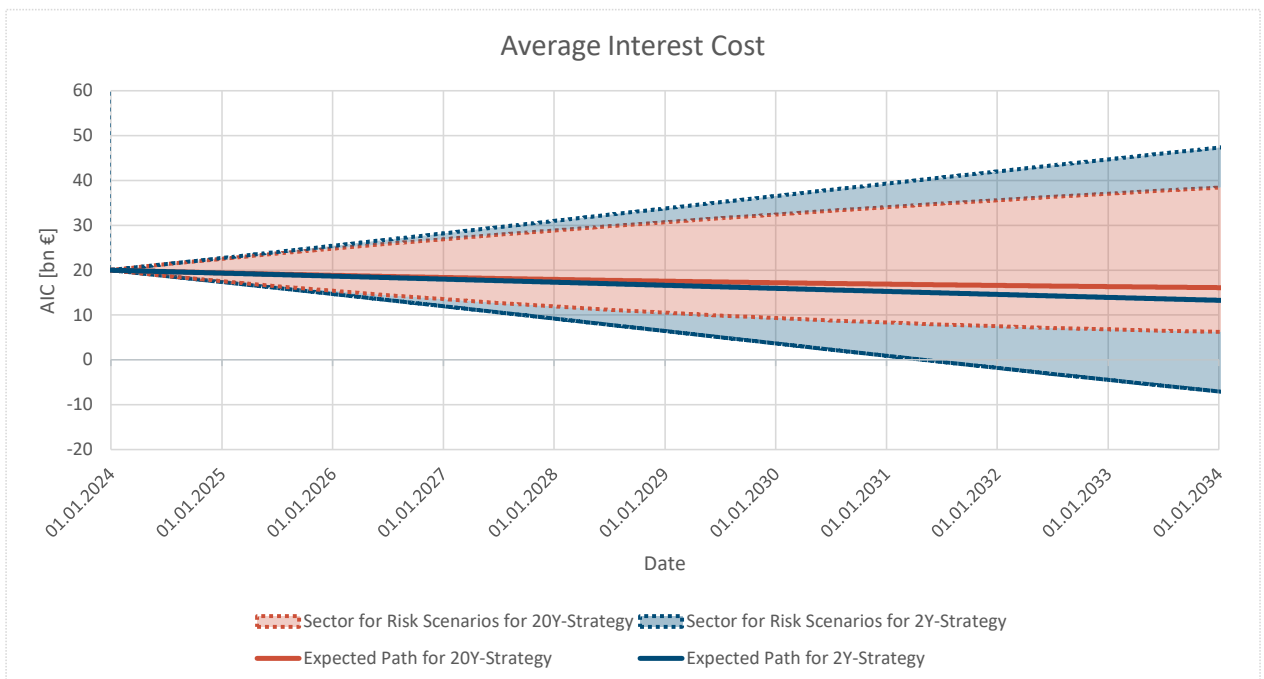


Figure 5: Average Interest Cost for the two different strategies under all three different yield scenarios shown in Figure 4.

Figure 5 shows the development of the AIC with starting date $t_0 =$ January 1st 2024 and an AIC-Horizon of $\Delta H = 15$ years for the two different considered strategies under the three selected yield scenarios. We



want to emphasize two main observations: The 2Y-Strategy is again more responsive to the yield development. In combination with the small downward shift in the scenario with the expected yield development this results in significantly lower AIC of 13.3 bn € for the 2Y-Strategy than the expected AIC of 16.1 bn € for the 20Y-Strategy.⁷

The second main observation is the significantly larger sector of the AIC for the 2Y-Strategy compared to the 20Y-Strategy. While the yield path with a 400 bp rise is a risk scenario, the AIC in this scenario can be interpreted as a Cost at Risk measure. We interpret the difference between the AIC on the expected path and the AIC on this risk scenario path as the AIC-Risk. With 34.0 bn €, the 2Y-Strategy implies a much larger risk than the 20Y-Strategy with an AIC-Risk of 22.3 bn €. This again shows the expected and desired behavior of shorter portfolios implying higher risk than longer portfolios. In combination these two observations lead to the typical trade-off for debt managers between cost and risk.

This analysis is based on very simple deterministic scenarios for illustration purpose only. However, it can be easily adapted to be based on e.g. sophisticated macro-economically motivated deterministic scenarios or a stochastic interest-rate model. Depending on the view, the AIC-Risk can be derived from the resulting distribution of the AIC under these different paths by using e.g. percentiles or the standard deviation. It is also possible to determine the AIC-Risk on a specific stochastic path due to market movements. Depending on the debt manager's actual point of interest, the AIC can provide a comprehensive measure for investigating future developments of the debt portfolio.

5.3 Portfolio Optimization and Decision Making

Based on the analysis presented in the previous section it is possible to illustrate a path to decision making in public debt management. With the combination of the AIC and its associated risk, a public debt manager can judge different considerable strategies similar to the standard portfolio selection approach developed by Markowitz (Markowitz, 1952). Figure 6 shows a cost-risk chart with the two specific strategies from the previous section as two examples of possible portfolio strategies. Additionally, we included a comparable 5Y-Strategy and a 10Y-Strategy, respectively, constructed with the same method. By rendering additional different possible portfolio strategies, calculating their cost and risk and including them in the cost-risk chart, or using optimization methods, it is possible to construct the efficient frontier. This efficient frontier indicates all portfolios that show minimal cost for a given risk, or minimal risk for a given cost. A rigorous treatment of the portfolio optimization routine is out of scope of this paper, and Figure 6 only shows a stylized efficient frontier. For a deeper treatment in the context of public debt management see e.g. (Bernaschi, et al., 2009), (Bergström, et al., 2002), (Bolder & Deeley, 2011), (Yavuz & Gölbaşı, 2019).

In practice, such an efficient frontier indicates the most reasonable portfolio strategies regarding AIC and AIC-Risk. The final decision about one specific strategy on (or near) the efficient frontier is dependent on the risk aversion of the debt manager: How much more AIC-Risk is acceptable for a given reduction of Average Interest Cost? Since the AIC fulfills all key requirements for public debt management, using AIC and AIC-Risk provides a comprehensive and intuitive way for weighing cost against risk.

⁷ In this one-factor model, the observation of lower cost for shorter strategies is enforced by the expected decline in yield rate. A more realistic yield model including the typical upwards sloping yield curve usually generates lower cost for shorter strategies even without a dedicated decline in yields.

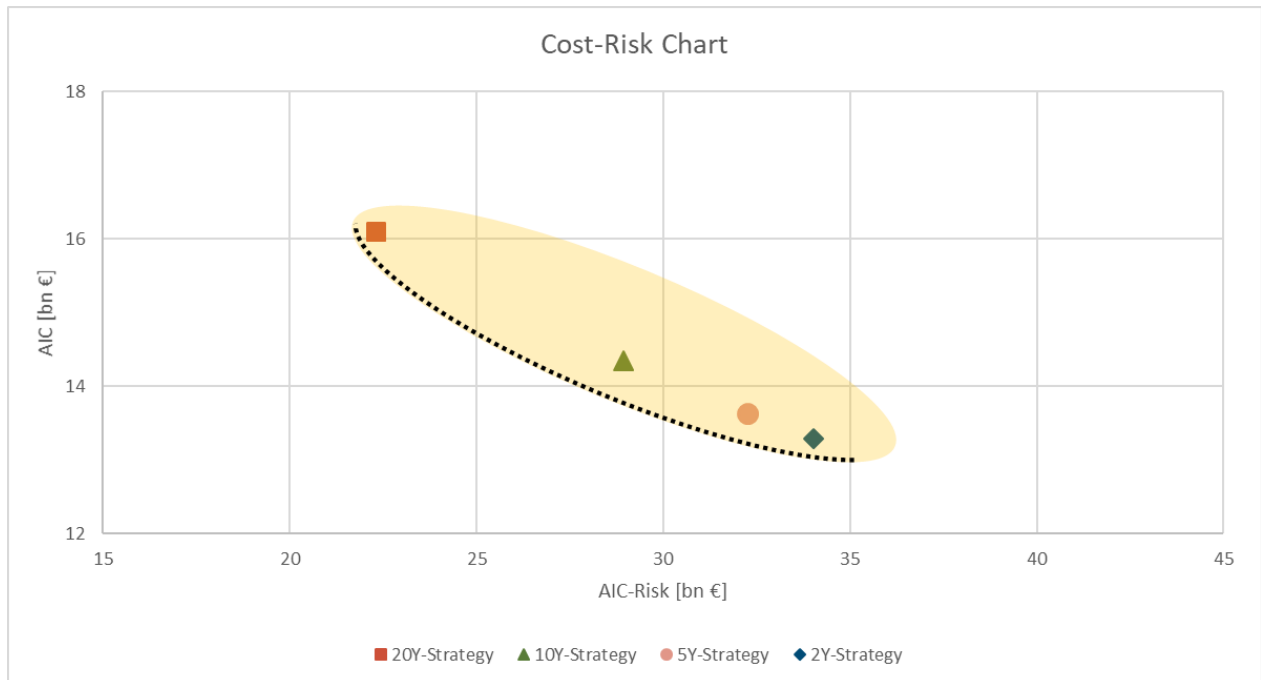


Figure 6: Cost-risk chart for decision-making with AIC-Risk and AIC for different strategies issuing different instruments. Additionally, the stylized area of all possible strategies and the resulting efficient frontier are indicated.

6 Summary and Outlook

In this article we discussed key features of a suitable cost and risk measure for public debt management and developed the novel measure Average Interest Cost and its corresponding risk. The Average Interest Cost is based on interest expenses over a long timespan including historic and expected future interest expenses. The expected future interest expenses can be interpreted as the cost associated with a hypothetical buyback of the current debt portfolio and refinancing the complete debt at current market conditions.

We showed that the AIC can accommodate all key requirements for public debt management including capturing every cashflow, a realistic perspective on long-lasting public debt, a high correlation with interest rates, an associated risk measure and lower risk for longer portfolios. With these properties, the Average Interest Cost improves established cost measures in public debt management. We concluded the paper with three analyses inspired by use cases at the German Finance Agency. These analyses include a historic performance analysis that can be used for benchmarking and reporting, a stochastic scenario analysis for evaluating future strategies, and a schematic illustration of a possible portfolio optimization for decision making. The list of possible use cases for AIC can be easily expanded to other issues of interest.

One possible modification to the presented AIC is to normalize the AIC by the present value or the debt of the portfolio and thereby construct the “Normalized Average Interest Cost”. The Normalized Average Interest Cost makes it trivial to compare diverse (sub)portfolios or distinct instrument classes consistently. However, this modification is beyond the scope of this paper and will be explored in a future publication.

The AIC is a highly condensed cost measure, which is useful to compare different portfolios and strategies from a long-term and sustainable perspective. While strategic decisions and high-level discussions can focus on this view, in practice it is still necessary to monitor more imminent aspects, e.g. budget constraints. Additionally, it can still be necessary to calculate the actual interest expenses expected for each year to avoid unbalances from year to year, which is not provided by the AIC.

The AIC-Risk based on a variance measure of the AIC is capturing refinancing risk driven by interest rate risk. While this is a major source of risk, other forms of risk like liquidity, credit, settlement, or operational



risk have to be monitored as well and treated separately, see e.g. (Lewis & Viñals, 2014). AIC-Risk is partly including rollover risk since the AIC incorporates the Refinancing Cost, but the concept is based on the ability to rollover the debt, albeit at possibly higher yields. The core of rollover risk, i.e. the inability to rollover the debt, is not encapsulated in the AIC and its risk and has to be monitored differently.

The main contribution of this novel cost measure is the extension of the widespread budgetary approach to cost in public debt management. This extension builds the bridge to a mark-to-market view based on the present value and combines the advantages of both approaches. With the AIC it is possible to generate a meaningful and economic complete metric as a base for different use cases. While the definition of the AIC is adjustable to fit specific needs of a public debt manager, the AIC provides a condensed cost measure that is easy to interpret, simple to calculate and that meets all key requirements for public debt management.

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Appendix A Derivation of Coupon Based View on AIC

In a very simplified view the sum of Market Value Compensation and Refinancing Cost corresponds to the already known fixed interest expenses and the expected cost of refinancing each single bond from the time of its maturity up to the end of the AIC-Horizon.

Assuming for simplicity, the debt portfolio consists of only one bond with nominal N , maturing at time t_M and coupon d . For a complete view of the accumulated future interest expenses up to the desired time $t_H = t + \Delta H > t_M$, the maturing bond has to be refinanced. This is done by a par coupon bond issued at time t_M maturing at time t_H , with the par coupon of c . The accumulated future interest expenses asserting a discrete yearly time grid can be expressed as

$$IE(t, t + \Delta H) = N \cdot (t_M - t) \cdot d + N \cdot (t + \Delta H - t_M) \cdot c.$$

Under the assumption of a completely flat yield curve, we can decompose the outstanding bond into a par coupon bond with coupon c , and the off-market part with coupon $(d - c)$:

$$\begin{aligned} IE(t, t + \Delta H) &= N \cdot (t_M - t) \cdot (d - c) + N \cdot (t_M - t) \cdot c + N \cdot (t + \Delta H - t_M) \cdot c \\ &= N \cdot (t_M - t) \cdot (d - c) + N \cdot \Delta H \cdot c. \end{aligned}$$

Note, that the second term is simply the cumulated interest expenses of a bond at current market conditions maturing at t_H , equivalent to the Refinancing Cost in the AIC. The first term, however, can be interpreted as the “undiscounted” Market Value Compensation in the AIC. This becomes clear by using the discount factor $DF(t, t_i)$, discounting from t_i to t , and rewriting the debt corresponding to the nominal of the outstanding bond

$$\begin{aligned} PV(t) - D(t) &= N \cdot \left(\sum_{t_i=t}^{t_M} d \cdot DF(t, t_i) + DF(t, t_M) - \left(\sum_{t_i=t}^{t_M} c \cdot DF(t, t_i) + DF(t, t_M) \right) \right) \\ &= N \cdot \sum_{t_i=t}^{t_M} (d - c) \cdot DF(t, t_i) \approx N \cdot (t_M - t) \cdot (d - c). \end{aligned}$$

Therefore, the sum of the Market Value Compensation and the Refinancing Cost can also be viewed (apart from discounting) as the accumulated future interest cost of the current portfolio and an appropriate refinancing of the maturing bonds up to the end of the horizon $t + \Delta H$.

Appendix B The Refinancing Yield in Detail

In 3.3 it is suggested to use the zero-yield with maturity at the end of the horizon as the refinancing yield, $\bar{z}(t, t + \Delta H) = z(t, t + \Delta H)$.

While this definition is exceptionally easy to calculate and might be sufficient for a lot of use cases, this definition makes the AIC highly correlated with a specific point of the yield curve. Therefore, in the



German Finance Agency we use a weighted average over the complete yield curve to also reflect changes of the yield curve structure into the AIC:

$$\bar{z}(t, t + \Delta H) = \frac{1}{2 \cdot \Delta H^2} \cdot \int_{\tau=0}^{2 \cdot \Delta H} \tau \cdot z(t, t + \tau) d\tau.$$

This specific choice also leads to a different interpretation: While this Refinancing Cost can still be seen as the cost of a single bond, just with an average yield, it can also be interpreted as a portfolio consisting of zero-bonds with different times to maturity from 0 to $2\Delta H$. In this interpretation the Refinancing Cost imply a linearly shrinking debt from current time point t until $t + 2\Delta H$.

Both above definitions of the Refinancing Cost assume that the “risk free” portfolio used for refinancing can be built with instruments based only on a single yield curve. This assumption makes the AIC reasonable simple and is appropriate for the German debt portfolio, where the overwhelming part of the portfolio consists of simple nominal bonds. The portfolio consists only to a small percentage of inflation linked bonds and green bonds, and no bonds in foreign currencies are issued.

For other public debt managers, it might be useful to modify the Refinancing Cost so that this part of the AIC is also sensitive to changes in exchange rates or inflation. With the general idea of a “risk-free” refinancing portfolio in mind, the AIC is easily adjustable to the specific need of the debt manager. For instance, the refinancing portfolio can be split into one part refinanced in local currency, and the other part in foreign currency.

Appendix C Modelling Details

The underlying model for the analyses presented in Chapter 5 is a highly simplified model of public debt management: For the considered timespan a constant debt of one trillion € has to be financed via different fixed bonds with different time to maturity. Each year, some of the outstanding bonds mature and have to be refinanced with a new bond, whose coupons are fixed at the then current rate. For simplicity we assume a flat one factor yield curve that also determines the refinancing yield for the AIC. In the historic analysis this flat yield curve is based on the actual 10-year rate of the German BUND yield curve. We also assume for simplicity that the date of redemption, date of issuance and the coupon payment date all fall on June 30th of the corresponding year.

Appendix C.1 Historic Analysis

For the historic analysis in 5.1, we focus on stable portfolios that are generated by a time-stable issuance strategy. Each year the same amount of each instrument is issued to finance the redeeming bonds. Two different strategies are compared here. The so called “2Y-Strategy” only consists of bonds with a maturity of 2 years at time of issuance, the so called “20Y-Strategy” in turn only consists of bonds with a maturity of 20 years at time of issuance. Both strategies are initialized at the December 28th 2012 at then par value. The imposed structure results in the 2Y-Strategy having to refinance 500 bn € each year, while the 20Y-Strategy only refinances 50 bn € each year. It becomes obvious that the 2Y-Strategy has a significantly lower average time to maturity compared to the 20Y-Strategy.

Figure 7 and Figure 8 show the development over time of the Market Value Compensation (i.e. the difference of present value and debt⁸) and the cumulated interest expenses for each of the two strategies. The cumulated interest expenses rise over time, with steps corresponding to the coupon-payments on June 30th each year. Although we assume a flat yield curve for all maturities, the two strategies generate different interest expenses over time. This emerges due to the different amount of refinancing each year

⁸ Since the debt of the portfolio is constant by construction, the Market Value Compensation essentially shows the movement of the *PV*, only shifted by the constant debt.



and therefore different portions of the portfolio being re-fixed at the then current rate. Note that the cumulated interest expenses for the 2Y-Strategy decrease in 2021 and 2022 because of the negative yields in 2020 and 2021 resulting in negative coupons for the issued bonds in that year. This cannot be observed for the 20Y-Strategy, since the fraction of negative coupons is too small to outweigh the bonds issued before with positive coupons.

The Market Value Compensation moves in opposite direction with the corresponding yield development, and also reacts to the coupon payment dates in opposite direction to the interest expenses. Note that the Market Value Compensation due to the close connection of the PV shows much larger movements for the 20Y-Strategy than for the 2Y-Strategy.

To calculate the AIC shown in Figure 2 in 5.1, the cumulated interest expenses and the Market Value Compensation have to be added to the Refinancing Cost. This Refinancing Cost is simply the product of the refinancing yield, the debt of 1 trillion and the horizon length of 15 years.

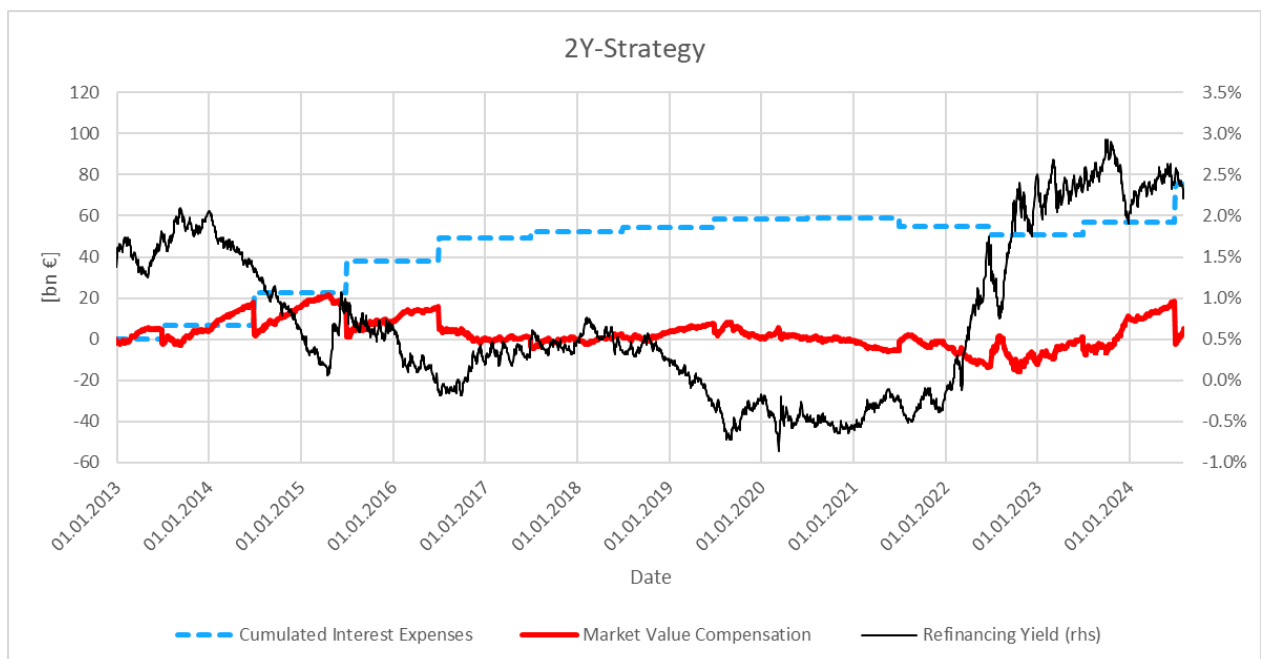


Figure 7: Development of the cumulated interest expenses and the Market Value Compensation for the 2Y-Strategy, and the refinancing yield.



Figure 8: Development of the cumulated interest expenses and the Market Value Compensation for the 20Y-Strategy, and the refinancing yield.

Appendix C.2 Future Analysis

The general setup for the future analysis in 5.2 is comparable to the method used for the historic analysis, described in Appendix C.1. Each strategy finances a constant debt of one trillion € by issuing new bonds each year, but using only 2Y-bonds or 20Y-bonds, respectively. However, the starting portfolio on January 1st 2024 is considered to be the same for both strategies. It consists of 100 billion € par bonds priced at a yield of 2.0% maturing in 2024, 2025, 2026, etc., respectively. Therefore, both strategies have to refinance 100 billion € each year due to the maturing initial portfolio. However, the 2Y-Strategy also has to refinance the nominal corresponding to the maturing newly issued bonds. This is not the case for the 20Y-Strategy, since the simulation horizon ends before the maturity of the first 20Y-bond. The setup with a common initial portfolio implies that both considered strategies diverge from each other relatively slowly.