Inflation-linked bonds are not a good instrument for financing government debt in emerging and developing economies: a view from risk-return analysis

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Extended Abstract

We investigate why inflation-linked (CPIL) instruments are not used much by governments of developing and emerging market economies despite many potential benefits of such instruments for lenders, borrowers as well as financial stability of the borrowing economies. We show theoretically and empirically that the limited utilization of CPIL instruments may owe to inferior risk-return characteristics of these instruments compared to other alternatives, e.g. in foreign currency (FCY) or in plain local currency (LCY). We find that CPIL instruments bring higher risk-adjusted borrowing costs than these alternative forms of borrowing, which may contribute to their lower use. Furthermore, we show that investors are more likely to be interested in CPIL instruments than borrowers, ceteris paribus. We show that CPIL instruments are not suitable for economies with strong control over the exchange rate and weak control of inflation. Neither are they suitable for countries with large volatility in the real exchange rate and terms-of-trade.

A. Introduction

There seem substantial benefits to investors, borrowers as well as local regulators from considering CPI-linked (CPIL) financing. For many developing and emerging market economies CPIL instruments may be a cheaper form of local currency (LCY) financing than fixed-rate LCY or foreign currency (FCY) instruments. For investors with a hard-currency balance sheet, CPIL instruments can offset the exchange risk involved in LCY lending with an inflation risk, thus providing for lower market risk premiums and more affordable and plentiful LCY financing. For borrowers, the interest risk in CPIL products may often be lower than in FCY financing (when expressed in an LCY equivalent) and can be hedged naturally through a link between the borrower's LCY revenues and inflation. For country authorities, CPIL financing is free from systemic financial stability risks involved in FCY borrowing, stimulates financial sector development and incentivizes a welfare-improving low inflation environment (Panizza and Taddei, 2020, Abbas and Rogoff, 2019).

Yet, despite all these benefits, CPIL financing is not used often. According to publicly available data on government debt, there have been only 346 issues of CPIL bonds in the past 15 years by 16 countries out of a total of 69 developing and emerging market economies in our sample.[4] [5] This contrasts with 13710 issues of FCY and LCY bonds issued by the same countries over the same period in total. Even the countries that have issued some CPIL have also issued FCY and plain LCY instruments during the same period.

In this paper, we investigate why the CPIL instruments are used so scarcely in financing the investments in developing and emerging market economies. We use a simple risk-return preference framework for analyzing the trade-offs facing the lender and borrower when considering CPIL instruments relative to the more conventional FCY or plain LCY instruments. We focus on the market risk the borrowers and lenders

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[4] There have probably been more CPIL loan instruments provided to the private sector in the EMEs over-the-counter. However, such data are not publicly available.
[5] We exclude developed economies from this version of the analysis.

take in these instruments and develop theoretical conditions under which CPIL loans can bring a cheaper source of LCY financing for developing economies than FCY and plain LCY instruments.

We show that in certain cases CPIL instruments can offset the exchange risk otherwise involved in LCY lending with an inflation risk, thus providing for lower market risk premiums and more affordable LCY financing. Specifically, we show that the benefits of CPIL for the borrower are most likely to arise in the environment with positive and high correlation between exchange rate depreciation and inflation, but low volatility in the real exchange rate. As a consequence, CPIL financing should work best in the economies with high business cycle volatility in inflation and exchange rate, but not necessarily commodity rich, because it provides protection against business cycle volatility, but not against shocks to terms-of-trade, real exchange rate and other fundamentals.

Perhaps surprisingly, we also find the economies with good inflation control are better suited for CPIL instruments than those with a high degree of exchange rate control, ceteris paribus. There is little reason to invest through CPIL instruments in credibly pegged currencies, because inflation is an unnecessary source of additional risk. LCY or FCY instruments provide safer alternatives.

However, our analysis also demonstrates that the CPIL borrowing will unlikely bring the same reward for the market risk borrowers take as the plain LCY or FCY alternatives. This is because such an adequate reward would require perfect correlation between inflation and nominal depreciation, which is a practical impossibility. We show that the risk-adjusted costs of CPIL instruments will always be higher than those of conventional FCY and LCY alternatives. As a result, the borrower has, other things being equal, always a greater incentive to borrow in LCY or FCY than through a CPIL instrument.

Furthermore, we find that the CPIL borrower can never be compensated for the market risk as well as the CPIL investor, putting a wedge between the incentives of both parties to engage in CPIL financing and making CPIL more attractive for the investor than the borrower.

We back these theoretical findings with empirical analysis of ex post macroeconomic data. Overall, we find robust support to our assertion that the risk-return characteristics given by the macroeconomic environment are an important factor in determining the attractiveness of CPIL financing. Most CPIL-issuing economies have met the inflation-exchange rate correlation condition. Only one out of 16 CPIL-issuing economies in our sample has not satisfied these conditions at all. Moreover, the group of CPIL bond issuing countries could have achieved much lower risk-adjusted costs of CPIL borrowing relative to FCY or LCY than the rest of the sample. Nevertheless, we also show that these risk-adjusted costs of CPIL borrowing would still have been higher than those of FCY borrowing or the risk-adjusted return achieved by the potential foreign investor.

Our research yields several important policy implications. One is that CPIL financing is unlikely to be a preferred way of borrowing in developing and emerging market economies irrespective of the ex-ante market risk expectations. This would require either a risk neutral borrower or some channel through which CPIL instruments would reduce some other than market risks facing the investor (e.g. the credit risk) and thus lead to an overall cheaper source of financing. Furthermore, CPIL instruments are not suitable for economies with strong control over the exchange rate and weak control of inflation. Neither are they suitable for countries with large volatility in the real exchange rate and terms-of-trade.

Several caveats are in place. Our analysis ignores any other possible motivations for issuing CPIL bonds than risk-return analysis. The practice is, of course, more colorful. Political considerations, debt management strategies of the governments/issuers and the investors' appetite for the credit, market or country risk in a particular period have a large influence on the type of instrument issued. Furthermore, we take into account only the market risk, ignoring other types of risks and premiums that may affect the choice of the instrument issued, e.g. the credit risk of the government, the liquidity risk or the convertibility/transfer risk.[6] On top of it, the interest rate risk is imperfectly measured using some short-term market benchmarks which may nonetheless not reflect well the marginal cost of LCY funding in the interbank market. Finally, our empirical analysis uses ex-post sample data and realized excess returns which may not reflect well the ex-ante expectations the investors had.

We intend to extend the present analysis in several directions. First, we are planning to expand the analysis to more countries, including from developed economies. Second, we would also like to consider and compare actual prices of LCY, FCY and CPIL instruments issued by governments in the past. This should give more insight into the actual Sharpe ratios that investors and borrowers have realized through these vehicles. Third, we would like to consider different maturities of the debt and analyze how the CPIL attractiveness may vary across different tenors. Fourth, we would like to extend the search for CPIL instruments through additional data sources.

B. Literature Review

Much of the theoretical research on CPIL instruments focuses on using CPIL in constructing an optimal portfolio. Theoretical analyses demonstrate that inflation-linked bonds should be included in an optimal portfolio, even when considering the presence of indexation lags. For instance, Li (2019) calculates the price of inflation-indexed bonds when payments are tied to a lagged price index, and determines the optimal bond portfolio considering both inflation and indexation lags. The research reveals a positive impact of indexation lags, typically viewed as a form of market friction, on the bond portfolio.

The empirical research on CPIL instruments assesses whether inflation-linked bonds outperform traditional bonds. The overarching conclusion is affirmative, but the studies generally do not consider whether the CPIL instruments present an adequate return relative to the market risks for the borrower and investor, and do not study the macroeconomic characteristics of the borrowing economy as a factor in determining the attractiveness of CPIL instruments.

For instance, Chopra, Mehta and Srivastava (2021) study inflation-linked bonds as a separate asset class and conclude that CPIL bonds offer advantageous excess returns compared to nominal bonds across different inflation scenarios. The inclusion of CPIL bonds in a portfolio enhances mean-variance efficiency, suggesting that these bonds can serve as both a hedge and a means to improve strategic asset allocation within a portfolio.

Furthermore, although there are findings available for developing markets, the majority of the empirical literature centers on developed markets. One of the exceptions is de Jong and Swinkels (2022) who focus on an American investor contemplating an allocation to emerging-market inflation-linked bonds instead of US bonds to obtain greater inflation compensation. The findings indicate that such an allocation would

[6] Some of these risks and the related risk premiums may correlate with the market risks considered in our analysis. For instance, the default risk may correlate with the exchange rate risk for countries with a large share of FCY debt.

have been beneficial over the past decade, even when considering latent risks like country defaults. Like us, they compute excess returns from investing into emerging market CPIL bonds using historical data, but they do not explicitly express the results in terms of risk-return characteristics that would compare CPIL instruments with other alternatives, e.g. FCY or plain LCY bonds. Unlike us, they ignore the risk-return perspective of the borrower.

Another branch of the empirical research employs the so-called "hypothetical indexed bonds approach". For instance, Auckenthaler, Kupfer, and Sendlhofer, R. (2015) employ a VAR model incorporating 3-month nominal T-bill rates and inflation rates to predict quarterly real interest rates. Using these predictions, they calculate hypothetical long-term yields for CPIL bonds and compare them to actual bond yields. The researchers then study how monetary policy and liquidity affect the CPIL bond yield curve in developed countries, but do not compare CPIL, FCY and plain LCY instruments.

C. Theoretical Analysis

We consider the risk-return preferences of a foreign investor to a developing or emerging market economy and a (sovereign) borrower from such an economy. We assume the investor is risk-averse and postulate that the preferences can be expressed using a specific value of the Sharpe ratio (SR), i.e. a ratio of an expected excess return over a risk-free return and the standard deviation of that excess return.

Furthermore, we assume that the only risk faced by the lender and the borrower involved in any forms of the investment is the market risk of interest and exchange rate movements in the borrowing economy. The lender's balance sheet is in FCY, while the borrower's is in LCY. Neither is naturally hedged against the market risk involved in the lending instrument between the two and we do not consider the liability side of the lender nor the asset side of the borrower.[7]

Overall, we consider three ways of lending to the borrower: in FCY, in LCY, and in CPIL. The respective risk-return characteristics of the three instruments for the investor are in each case as follows (see Math Appendix, Foreign Investor Characteristics): [8]

Table 1. Risk-return characteristics of the different instruments for the investor

Investor:	FCY	LCY	CPIL
Lending interest rate	$i^*_{t,h}$	${i_{t,h}}^{LCY}$	${i_{t,h}}^{LCY,CPIL}$
Excess return	$ER^{Inv,FCY}_{t,h} = 0$	$ER^{Inv,LCY}_{t,h} = E\Delta_{\mathrm{h}}S_t + \lambda_t - \Delta_{\mathrm{h}}S_t$	$ER^{Inv,LCY,CPIL}_{t,h} = \pi_t + \omega_t - \Delta_{\mathrm{h}}S_t - i^*_{t,h}$
Expected Excess Return	$EER^{Inv,FCY}_{t,h} = 0$	$EER^{Inv,LCY}_{t,h} = \lambda_t$	$EER^{Inv,LCY,CPIL}_{t,h} = E\pi_t + \omega_t - E\Delta_{\mathrm{h}}S_t - i^*_{t,h}$

[7] This assumption is especially important for the position of the borrower who is by definition naturally hedged against the inflation risk, as the tax revenues are collected in local currency. Furthermore, one could naturally suppose that a developing country borrower will have much of the revenues denominated in FCY too. We deliberately abstract from these considerations, because they would automatically render CPIL instruments free of any market risk for the borrower, making them very attractive. Yet, this runs against the empirical observation that CPIL instruments are not much used. Instead, we picture a debt management office of the borrower choosing the borrowing instruments on their own merits, without considering the capacity to repay them.

[8] All interest rates and rates of change are expressed as annualized rates accruing over the same period with simple compounding using log-approximations. In the main text we suppress the time subscripts and other variable notation where unambiguous to ease the exposition. The mathematical appendix provides full notational detail on the variables and equations used.

Standard error of the Excess return	$\sigma\left(ER_{t,h}^{Inv,FCY}\right) = 0$	$\sigma\left(ER_{t,h}^{Inv,LCY}\right) = \sigma^{\Delta_h S}$	$\sigma\left(ER_{t,h}^{Inv,LCY,CPIL}\right) = \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}}$
Sharpe Ratio	NA	$SR^{Inv,LCY} = \frac{\lambda_t}{\sigma^{\Delta_h S}}$	$SR^{Inv,LCY,CPIL} = \frac{E\pi_t + \omega_t - E\Delta_h S_t - i_{t,h}^*}{\sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}}}$

Source: Authors' calculations

The investor's risk-free investment is an investment in FCY, because that way the investor does not face any market risk. By contrast, both the LCY and CPIL investments carry market risk for the investor: the LCY through exchange rate movements (Gadanecz et al., 2014, Leung and Wan, 2019), and CPIL through both the exchange rate and inflation movements.

Because of the risk involved, the investor must be compensated by higher expected excess returns when investing in CPIL or LCY instruments in order to be indifferent among the three forms of investing. We postulate that the expected excess return in either CPIL or LCY investment must be such, so as to lead to a Sharpe ratio equal to a specific positive value of $\overline{SR} > 0$.[9]

Altogether, this implies the following conditions for the lending rates in each three instruments:

Table 2. Lending rates

	FCY	LCY	CPIL
Lending interest rate	$i_{t,h}^*$	${i_{t,h}}^{LCY} = E\Delta_h S_t + i_{t,h}^* + \lambda$	${i_{t,h}}^{LCY,CPIL} = \pi_t + \omega$

Source: Authors' calculations

where $\omega \geq 0$ and $\lambda \geq 0$ are constant risk premiums required by in the investor when investing in the CPIL or LCY instrument respectively such that the Sharpe ratio is in each case equal to $\overline{SR}$.[10] If the investor is risk neutral, then $\omega = \lambda = 0$. For instance, the LCY market premium λ is determined so that

$$SR^{Inv,LCY} = \frac{\lambda}{\sigma^{\Delta_h S}} = \overline{SR} \Rightarrow \lambda = \overline{SR}\ \sigma^{\Delta_h S}.$$

The CPIL market premium ω is determined analogously from:

$$SR^{Inv,LCY,CPIL} = \frac{E\pi_t + \omega_t - E\Delta_h S_t - i_{t,h}^*}{\sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}}} = \overline{SR}$$

We now consider the point of view of a (sovereign) borrower from a developing or emerging market economy. For such a borrower, the market-risk-free form of borrowing is in LCY. FCY borrowing carries an exchange rate risk and CPIL carries both the exchange rate and inflation risks. We therefore define the expected excess return (and Sharpe ratio) for the borrower as the difference between the risk-free rate of i^{LCY} and the respective expected interest cost in each form of investment. In other words, the expected costs of borrowing in CPIL or FCY must be lower than the risk-free rate of i^{LCY} so that a risk-averse borrower considers them.

The risk-return characteristics of the three types of borrowings are in Table 3 (see Math Appendix, Domestic Borrower Characteristics).

[9] For the purpose of this analysis, we ignore other sources of the investor's profit than the reward for risk.

[10] For the ease of exposition, we define risk premiums as additive, which is in line with common investment practice. Similar results could be obtained for multiplicative risk premiums.

Table 3. Risk-return characteristics of the different instruments for the borrower

Borrower:	FCY	LCY	CPIL
Borrowing interest rate	$i^*_{t,h}$	${i_{t,h}}^{LCY}$	${i_{t,h}}^{LCY,CPIL}$
Excess return	$ER^{Bor,FCY}_{t,h} = {i_{t,h}}^{LCY} - i^*_{t,h} - \Delta_{\mathrm{h}} S_t$ $= E\Delta_{\mathrm{h}} S_t + \lambda_t - \Delta_{\mathrm{h}} S_t$	$ER^{Bor,LCY}_{t,h} = 0$	$ER^{Bor,CPIL}_{t,h} = {i_{t,h}}^{LCY} - {i_{t,h}}^{LCY,CPIL}$ $= E\Delta_{\mathrm{h}} S_t + i^*_{t,h} + \lambda_t - \pi_t - \omega_t$
Expected Excess Return	$EER^{Bor,FCY}_{t,h} = \lambda_t$	$EER^{Bor,LCY}_{t,h} = 0$	$EER^{Bor,CPIL}_{t,h} = E\Delta_{\mathrm{h}} S_t + i^*_{t,h} + \lambda_t - E\pi_t$ $- \omega_t$
Standard error of the Excess return	$\sigma\left(ER^{Bor,FCY}_{t,h}\right) = \sigma^{\Delta_{\mathrm{h}} S}$	$\sigma\left(ER^{Bor,LCY}_{t,h}\right) = 0$	$\sigma\left(ER^{Bor,CPIL}_{t,h}\right) = \sigma^{\pi}$
Sharpe Ratio	$SR^{Bor,FCY} = \frac{\lambda_t}{\sigma^{\Delta_{\mathrm{h}} S}}$	NA	$SR^{Bor,CPIL} = \frac{E\Delta_{\mathrm{h}} S_t + i^*_{t,h} + \lambda_t - \pi_t - \omega_t}{\sigma^{\pi}}$

Source: Authors' calculations

Risk-neutral borrower

First, we consider the case of a risk-neutral borrower. The risk-neutral borrower will choose the instrument with the lowest expected borrowing costs. In order for the CPIL to be a preferred alternative to an LCY borrowing, it must be that the CPIL is cheaper in expectations, i.e., that the excess return for the borrower is positive:

$$EER^{Bor,CPIL}_{t,h} > 0$$

We show in the Math Appendix (Propositions 1 and 2) that if the excess return for the CPIL borrower is positive, then it is also higher than the expected return from FCY borrowing, i.e. borrowing in CPIL is cheaper relative to both LCY and FCY alternatives:

$$EER^{Bor,CPIL}_{t,h} > 0 \Rightarrow EER^{Bor,CPIL}_{t,h} > EER^{Bor,FCY}_{t,h}$$

Elaborating on the $EER^{Bor,CPIL}_{t,h} > 0$ condition further we see:

$${i_{t,h}}^{LCY,CPIL} < {i_{t,h}}^{LCY}$$

$$E\pi_t + \omega_t < E\Delta_{\mathrm{h}} S_t + i^*_{t,h} + \lambda_t$$

$$E\pi_t + \omega_t - E\Delta_{\mathrm{h}} S_t - i^*_{t,h} < \lambda_t$$

This is equivalent to $EER^{Inv,LCY,CPIL}_{t,h} < EER^{Inv,LCY}_{t,h}$. In other words, CPIL will be a cheaper source of borrowing if and only if it earns a lower expected return for the investor than a plain LCY instrument.

This may seem a paradox, but it makes sense, if the volatility of the investor's returns from a CPIL instrument is lower than that from a plain LCY instrument, because the investor can then charge a lower risk premium on the CPIL instrument than the LCY instrument. Indeed, assuming a generally real-averse investor with a positive SR, the Proposition 3 in the Math Appendix shows that

$$EER^{Inv,LCY}_{t,h} > EER^{Inv,LCY,CPIL}_{t,h}$$

$$\left(\sigma^{\Delta_{\mathrm{h}} S}\right)^2 > (\sigma^{\pi})^2 + \left(\sigma^{\Delta_{\mathrm{h}} S}\right)^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}} S}$$

$$\rho > \frac{1}{2}\frac{\sigma^{\pi}}{\sigma^{\Delta_{\mathrm{h}}S}} > 0 \qquad (1)$$

In other words, for the CPIL to be the preferred form of borrowing for the risk neutral borrower, the correlation between inflation and exchange rate depreciation must be positive and sufficiently high so as to offset the effect of exchange rate and inflation volatility on the CPIL returns for the investor. If the correlation is negative, then the real exchange rate is so volatile that there is no such offset: the CPIL returns of the investor are more volatile than the sum of inflation and depreciation volatility.

This makes intuitive sense. Because the investor requires compensation for the market risk, investors can charge lower market risk premiums for CPIL than on otherwise equivalent LCY instruments, if the returns from the former are expected to be less volatile (uncertain) than the latter. The excess return under the CPIL instrument is subject to both inflation and exchange rate risks, whereas the excess return on an LCY instrument is subject to the exchange rate risk only. But the overall risk in the CPIL returns can still be smaller than the exchange rate risk involved in plain LCY investing, if the movements in inflation correlate well with those of nominal depreciation, thus offsetting the impact of the exchange rate movements on the investor's CPIL returns. This creates conditions for charging lower market premiums and making CPIL instruments cheaper in expectations for borrowers than plain LCY instruments.

Condition (1) has an important interpretation for the macroeconomic situation and monetary policy of the borrowing economy. Note that the condition is equivalent to (Math Appendix, Proposition 3):

$$\left(\sigma^{\Delta_{\mathrm{h}}S}\right)^2 > \left(\sigma^{\Delta Z}\right)^2 \equiv (\sigma^{\pi})^2 + \left(\sigma^{\Delta_{\mathrm{h}}S}\right)^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}}S}$$

where ΔZ refers to real exchange rate depreciation. In other words, real depreciation variance is equivalent to the variance of the lender's return on a CPIL instrument (see Table 1). The condition says that the variance of nominal depreciation must be higher than that of real depreciation, which also implies positive correlation between inflation and nominal depreciation. Economies exhibit positive correlation between inflation and nominal depreciation if and only if the variance of real depreciation is smaller than the sum of inflation and nominal depreciation variances (see Math Appendix, Correlation Analysis).

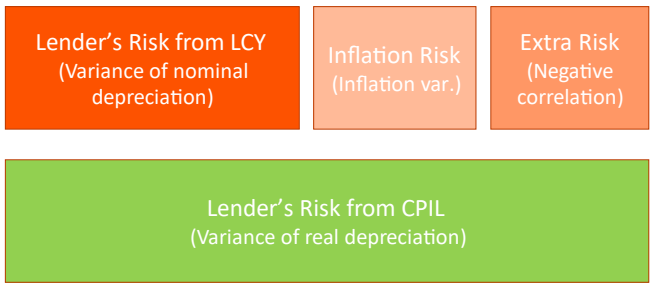
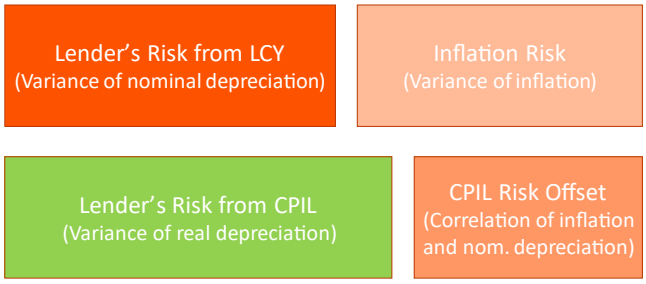
We can therefore categorize the macroeconomic conditions of an economy for our purposes by the relative size of real depreciation variance. Exhibit 1 below distinguishes among three important cases. In Case 1, real depreciation variance is larger than the sum of inflation and nominal depreciation variances. This can occur only if nominal exchange rate depreciation and inflation are negatively correlated. Such a situation may arise in economies exposed to frequent and significant shocks to terms-of-trade or other productivity levers, such as in economies rich or poor in natural resources. Such economies do not satisfy Condition (1).

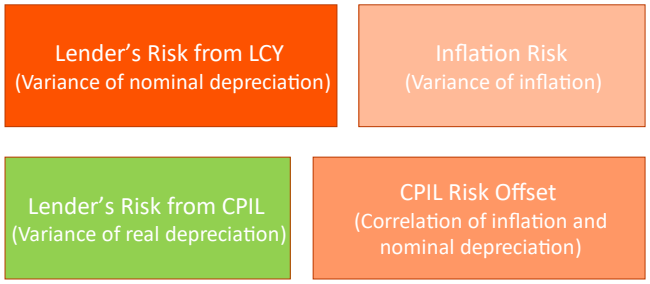
In Case 2, real depreciation variance no longer dominates the sum of inflation and nominal depreciation variances, because inflation and nominal depreciation are positively correlated. The positive correlation is a sign of typical business cycle shocks, including fiscal and monetary policy shocks, to which inflation and depreciation tend to react together (with some lags). The positive correlation reduces the size of the real depreciation variance, but not enough – the real depreciation variance is still higher than the nominal depreciation variance. Such economies do not satisfy Condition (1) either.

Finally, in Case 3, the correlation between inflation and nominal depreciation is so strongly positive that the variance of real depreciation is smaller than the variance of nominal depreciation. Such economies are primarily exposed to business cycle and policy shocks; frequent shocks to productivity or terms of trade do not take place. Only such economies satisfy Condition (1).

In summary, Condition (1) implies that CPIL instruments should be more affordable (and therefore more in demand) in economies with high business cycle volatility characterized by co-movements of inflation and nominal depreciation, but not in economies in which shocks to the real exchange rate or terms-of-trade are prominent. Such shocks are often characterized by negative correlation between depreciation and inflation and are likely to be important in economies either very rich or poor in natural resources.

Exhibit 1: Classification of macroeconomic conditions suitable for CPIL instruments

Case 1: Unsuitable for CPIL borrowers *Variance of lenders' returns from CPIL instruments is higher than the sum of variances of nominal depreciation and inflation, if and only if correlation between inflation and ex rate depreciation is negative. Such a situation may arise, for intance, if the economy faces mostly terms-of-trade and other real exchange rate shocks.*	
Case 2: Unsuitable for CPIL borrowers *Variance of lenders' returns from CPIL instruments is higher than variance of returns from LCY lending. Correlation between inflation and ex rate depreciation is not sufficient to offset the effect of inflation volatility on lenders' returns from CPIL instrument.*	

Case 3: Suitable for CPIL borrowers *Variance of lenders' returns from CPIL instruments is lower than variance of returns from LCY lending. Inflation and exchange rate depreciation are highly correlated, as business cycle, fiscal and monetary policy shocks are prevalent.*	

Source: Authors

Perhaps surprisingly, Condition 1 also implies that economies with good inflation control (σ^{π} small) will be better candidates for CPIL instruments than those with a high degree of exchange rate control ($\sigma^{\Delta S}$ small), ceteris paribus. This too has an intuitive interpretation: there is little reason to invest through CPIL instruments in credibly pegged currencies, because inflation is an unnecessary source of additional risk. LCY or FCY instruments provide safer alternatives and can therefore be priced more favorably from the perspective of the borrower.

By contrast, good inflation control means that CPIL instruments do not need a high-risk margin for the inflation risk. This could perhaps partly explain why CPIL instruments are issued by developed economies, e.g., the US and UK, with good inflation control. Moreover, developed economies are less likely to be subject to real exchange rate and terms-of-trade shocks than to business cycle shocks.

Risk-averse borrower

Let's now consider a risk-averse borrower. Such a borrower will weigh the expected costs from CPIL and FCY borrowings against the risk associated with these instruments. Such an investor will prefer CPIL bonds if the respective SR (risk-adjusted costs) is greater (are lower) than the SR (risk-adjusted costs) of the FCY borrowing.

In particular, note from Table 1 and Table 3 that the SR of the borrower in the case of FCY borrowing is the same as the SR of the investor in the case of LCY borrowing. In other words, LCY and FCY forms of investments represent two polar cases of who carries the exchange rate risk and the related compensation. In LCY investment, the exchange rate risk is borne by the investor and the borrower must therefore pay a risk premium to the investor which makes the LCY borrowing myopically (in a risk-neutral setting) more expensive than FCY borrowing. By contrast, in FCY investment, the exchange rate risk is borne by the borrower, which makes the borrowing rates myopically cheaper than LCY rates. This way the borrower in FCY indirectly receives the risk premium that would have gone to the investor in LCY borrowing. In relation to risk, however, both forms of investments are equivalent for both the investor and the borrower.

We have reached an important conclusion: the risk-adjusted costs for the FCY borrower and the risk adjusted return for the investor in plain LCY or CPIL are equal:

$$SR^{Bor,FCY} = SR^{Inv,CPIL} = SR^{Inv,LCY} = \overline{SR}$$

What remains to be determined is the relation of the risk adjusted cost (SR) of the borrower in the case of CPIL borrowing. First, the Math Appendix (Corollary 1 and Proposition 4) shows that

(i) Higher correlation between depreciation and inflation increases the borrower's risk-adjusted excess return from a CPIL bond (reduces the risk adjusted costs).

(ii) $SR^{Bor,CPIL} = \overline{SR}$ iff inflation and FX depreciation are perfectly correlated ($\rho = 1$).

As the latter condition is a practical impossibility, the relation between $SR^{Bor,CPIL}$ and $\overline{SR}$ is determined by the ratio between standard deviations of exchange rate depreciation and inflation $k = \frac{\sigma^{\Delta S}}{\sigma^{\pi}}$. The Math Appendix (Propositions 5 and 6) shows that

(iii) $\frac{SR^{Bor,CPIL}}{\overline{SR}}$ increases with k

(iv) $SR^{Bor,CPIL} \leq \overline{SR}$ for all values of k and ρ.

These conclusions confirm the intuition gained when studying the risk-neutral borrower: countries with a high degree of exchange rate management and/or countries with a high incidence of real exchange rate shocks (low correlation between inflation and exchange rate) are not set to benefit from CPIL instruments. Plain FCY and LCY instruments are likely to have a better risk-return profile for the borrower.

On the other hand, for the borrowers to benefit from CPIL instruments, not only must the exchange rate be very flexible, but also their control over inflation must be relatively good and the prevailing nature of macroeconomic shocks be such that inflation and depreciation are highly correlated.

Such conditions making CPIL a preferred borrowing instrument may not be met by every emerging market or developing economy. Some of them engage in heavy exchange rate management (e.g., to keep monetary control or to prevent the LCY value of FCY debt from spiraling out of control) and their inflation control is poor. At the same time, some of these countries are either commodity rich or, by contrast, dependent on commodity imports, making terms-of-trade and real exchange rate an important source of macroeconomic volatility so that correlation between inflation and depreciation is low.

The most important conclusion, however, is that the risk-return characteristics of CPIL instruments can never be better for the borrower than those of an FCY instrument. The SR of the borrower borrowing through CPIL will always be weakly lower than the SR the borrower can obtain from an FCY borrowing. In other words, the risk adjusted costs of borrowing in CPIL will always be higher than the risk-adjusted costs of borrowing in FCY (Math Appendix, Corollary 2):

$$SR^{Bor,CPIL} \leq SR^{Bor,FCY} = SR^{Inv,CPIL} = SR^{Inv,LCY} = \overline{SR}$$

Furthermore, the SR of the borrower in CPIL will also be lower than the SR of the investor in CPIL. The CPIL borrower can never be compensated for the risk as well as the CPIL investor. This puts a wedge between the incentives of the investor and borrower to engage in CPIL instruments – the investor is likely to prefer CPIL instruments more than the borrower.

Summary of predictions

Our key theoretical predictions regarding the conditions for the issuance of CPIL instruments can be summarized as follows:

Prediction 1: Countries issuing CPIL satisfy condition (1), i.e., correlation between nominal depreciation and inflation is positive, while volatility of nominal depreciation is low relative to that of inflation, so that the variance of real depreciation is smaller than that of the nominal depreciation;

Prediction 2: Countries with lower risk-adjusted costs of CPIL borrowing (higher CPIL SRs) relative to FCY or LCY borrowing are more likely to issue CPIL bonds than otherwise;

Prediction 3: The risk-adjusted costs of CPIL borrowing will always be higher than those of conventional FCY alternatives;

Prediction 4: Risk-averse borrowers will never find the CPIL instruments as attractive as risk-averse foreign investors.

D. Empirical analysis

We now examine the empirical relevance of our key theoretical findings on a set of 69 emerging market and developing economies over the period from 2000 to 2024. A full list of the countries, along with the data sources, can be found in the data appendix. The exact estimation range for each country varies and depends on the availability of reliable and comparable economic data for the exchange rate, inflation and the interest rate. For the CPIL issuing countries those time ranges include the periods during which they issued those securities.[11]

In our analysis we take that ex-post historical macroeconomic data represent true ex-ante expectations of investors and borrowers about these variables during the particular estimation periods. This is, of course, a huge simplification, but a good starting point for the analysis.[12] We use these macroeconomic characteristics to compute ex-post returns and Sharpe ratios of various instruments that could have been hypothetically expected by the lenders and borrowers during the relevant periods under perfect foresight. In so doing we abstract from any transaction costs as well as other premiums that foreign lenders might have reasonably expected to receive as compensation for other than market risks, e.g., liquidity premiums, convertibility and transfer premiums etc.

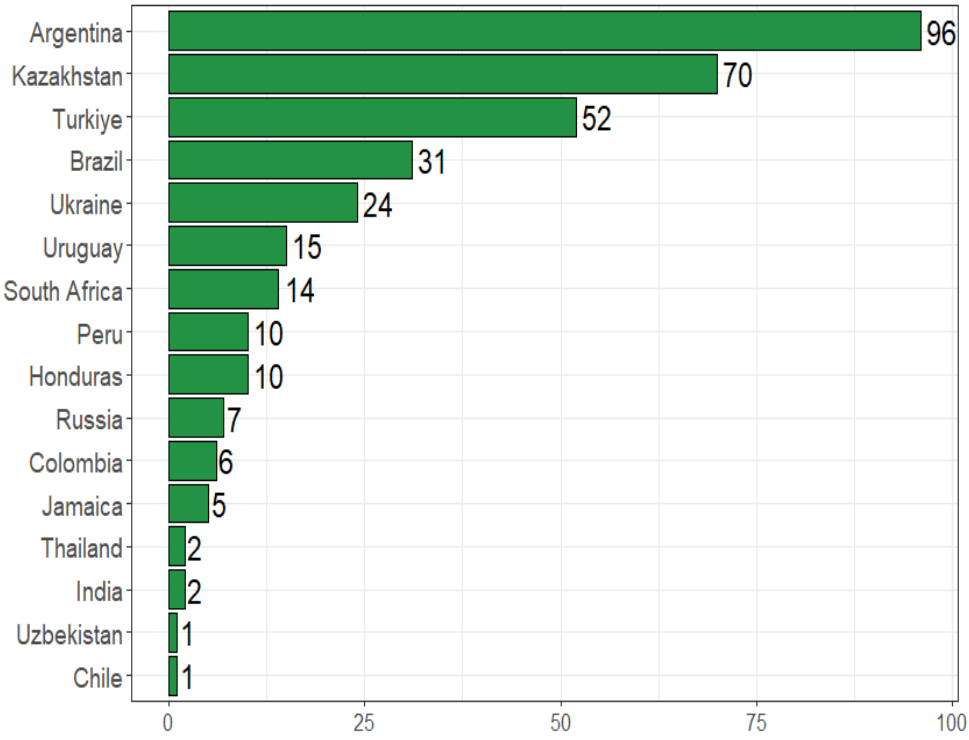
In computing standard errors and excess returns from macroeconomic data we use three different horizons of 1-month, 12-month, and 36-month respectively to test the robustness of our results.

Out of the 69 emerging and developing countries under consideration we have identified that in only 16 of them the governments have issued some form of a CPIL instrument (bond). The total number of CPIL issues is thus 346. The leading issuers have been Argentina, Kazakhstan and Turkey (Figure 1).

[11] Variations of the estimation period around the period during which the countries had the CPIL bonds outstanding did not produce qualitatively different results. As some countries (e.g. Uzbekistan) experienced enormous depreciation spells during these periods, we have also tested the robustness of the results to removing such outlier spells from the sample.

[12] Further work in this respect is underway.

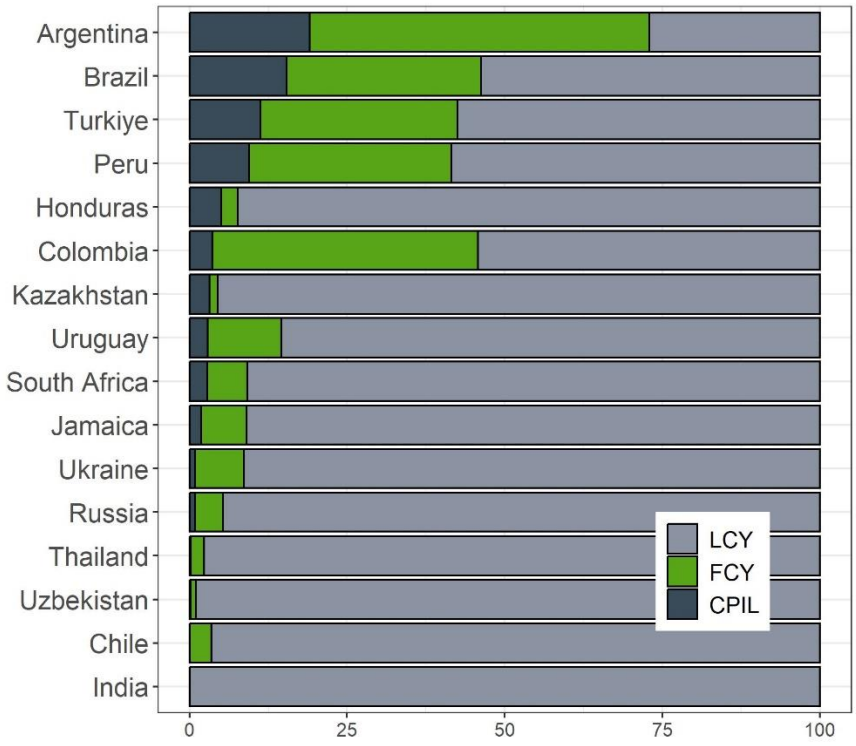
Figure 1. Number of bonds issued by the countries in 2015-2024



Source: Cbonds database

This stands in contrast with the issuance of other forms of government debt – issued in or indexed to FCY and plain LCY debt – which have been far more frequent (Figure 2). In terms of proportions, Argentina stands out as the most active user of CPI-linked bonds, with approximately one third of its issued bonds being CPI-linked.

Figure 2. Structure of bond issuance by instrument



Source: Cbonds database

Prediction 1: Countries issuing CPIL satisfy condition (1), i.e. correlation between nominal depreciation and inflation is positive, while volatility of nominal depreciation is low relative to that of inflation, so that the variance of real depreciation is smaller than that of the nominal depreciation.

Figure 3 depicts the correlation coefficient between inflation and nominal depreciation over the horizon of 12. The chart for the 1-month and 36-month horizons can be found in the appendix.

We observe that 41 out of 69 countries have the correlation positive on all three horizons. For many countries, at least one of the correlation coefficients is quite high, close to unity.

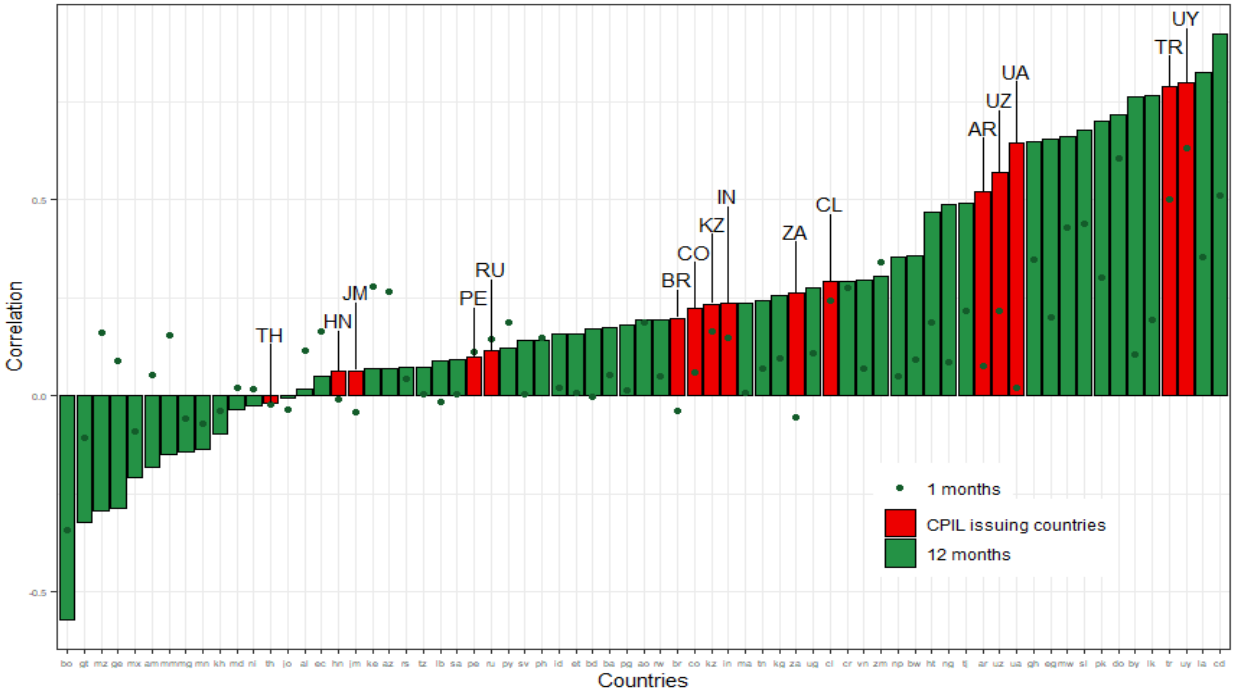
Higher correlation makes CPIL more attractive for the borrower, as the movements in inflation tend to compensate exchange rate changes, thus commanding lower risk premiums from investors. Indeed, we find that, with the exception of Thailand, all CPIL issuing countries have had the correlation positive on at least one of the horizons and ten of them had it positive on all three horizons. For 36-month correlations, Turkey, Uruguay and Ukraine have correlations around 0.8; others have it below 0.5.[13]

By contrast, 7 countries had the correlation negative on all three horizons and 21 countries had the correlation negative on at least one of the horizons. As such, these countries do not satisfy Condition (1)

[13] The 36-month correlations are in general higher than the 1-month correlations, as expected given the time lags involved in the exchange rate pass-through to inflation (and vice versa). The median correlation is 0.09 for the 1-month period, increasing to 0.2 for the 36-month period. Out of the CPIL issuing countries, only Kazakhstan and Russia demonstrate higher correlation on the 1-month than on the 36-month horizon.

on the respective horizons, belonging to Case 1 from the previous section. Negative correlation is a symptom of real exchange rate shocks and makes CPIL less attractive for the borrower. Borrowers from such economies are not likely to profit from CPIL borrowing relative to FCY or plain LCY borrowing. Only one CPIL issuing country in our sample (Thailand) had negative correlation between inflation and exchange rate depreciation on all three horizons, i.e., no volatility offset between inflation and exchange rate depreciation.[14]

Figure 3. Correlation coefficients between inflation and changes in the nominal exchange rate



Source: Authors' calculations

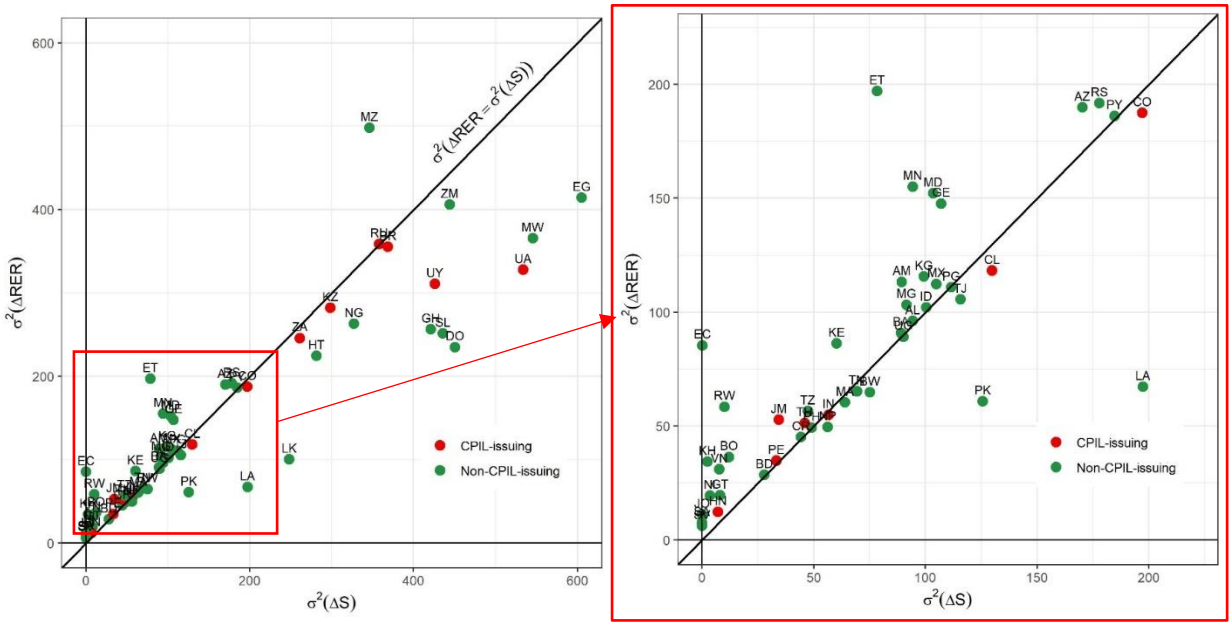
As we assert in the previous section, positive correlation between inflation and exchange rate depreciation is only a necessary, but not a sufficient condition for a CPIL instrument to be attractive for the risk-neutral borrower. The correlation must be high enough to ensure that the variance of real depreciation is lower than the variance of nominal depreciation. Only then will the expected costs for the CPIL borrower be lower than those involved in the plain LCY borrowing.

Putting it all together, Figure 4 displays which countries in our sample satisfy condition (1) using the comparison of standard errors between nominal and real depreciation (graphically it corresponds to the area below the 45-degree line) on the 12-month horizon. We see that only about half of our sample (31 countries) has met the condition for this horizon.[15] If we compare this list with the list of countries that have issued at least one CPIL instrument during this period (they are highlighted in red in the chart), we see that, with the exception of four (Jamaica, Honduras, Thailand and Peru), all the CPIL issuing

[14] 6 CPIL issuing countries had the correlation negative on at least one of the horizons.
[15] 32 countries on 1-month and 28 countries on 36-month horizons respectively.

economies have satisfied condition (1) and therefore had a positive excess return (and SR) from issuing CPIL bonds on the 12-month horizon.

Figure 4.Sample standard errors of real and nominal exchange rate for 12-month horizon[16]



Source: Authors' calculations

At the same time, it is interesting to note, that 24 economies in our sample have the correlation between inflation and exchange rate positive on the 12-month horizon, but did not satisfy condition (1).[17] Those belong to Case 2 from the previous section: inflation and depreciation movements partially compensated one another, but not sufficiently to make CPIL borrowing cheaper than the plain LCY alternative. Honduras, Jamaica, Russia and Peru are among those countries – the countries that issued CPIL bonds and had the correlation positive on the 12-month horizon, but did not satisfy condition (1).

Overall, Table 2 shows the split of the countries by the cases identified in Exhibit 1 when Condition 1 is evaluated on the three horizons. We see that the number of countries satisfying Condition 1 is relatively uniform (more than 40% of the sample) across the horizons.

[16] Myanmar, Angola, Turkey and Belarus not shown in the chart due to the large historical exchange rate volatility.
[17] 23 countries on 1 month, and 20 on 36 month.

Table 2: Split of the countries by the cases identified in Exhibit 1

			Case 1: Unsuitable for CPIL borrowers (Condition 1 not satisfied)	**Case 2: Unsuitable for CPIL borrowers (Condition 1 not satisfied)**	**Case 3: Suitable for CPIL borrowers (Condition 1 satisfied)**
Horizon			*Variance of lenders' returns from CPIL instruments is higher than the sum of variances of nominal depreciation and inflation, if correlation between inflation and ex rate depreciation is negative. Such a situation may arise, for instance, if the economy faces mostly terms-of-trade and other real exchange rate shocks.*	*Variance of lenders' returns from CPIL instruments is higher than variance of returns from LCY lending. Correlation between inflation and ex rate depreciation is not sufficient to offset the effect of inflation volatility on lenders' returns from CPIL instrument.*	*Variance of lenders' returns from CPIL instruments is lower than variance of returns from LCY lending. Inflation and exchange rate depreciation are highly correlated, as business cycle, fiscal and monetary policy shocks are prevalent.*
1 month	# of countries	**69**	**14**	**23**	**32**
	% of countries	*100%*	*20%*	*33%*	*46%*
12 months	# of countries	**69**	**14**	**24**	**31**
	% of countries	*100%*	*20%*	*35%*	*45%*
36 months	# of countries	**69**	**21**	**20**	**28**
	% of countries	*100%*	*30%*	*29%*	*41%*

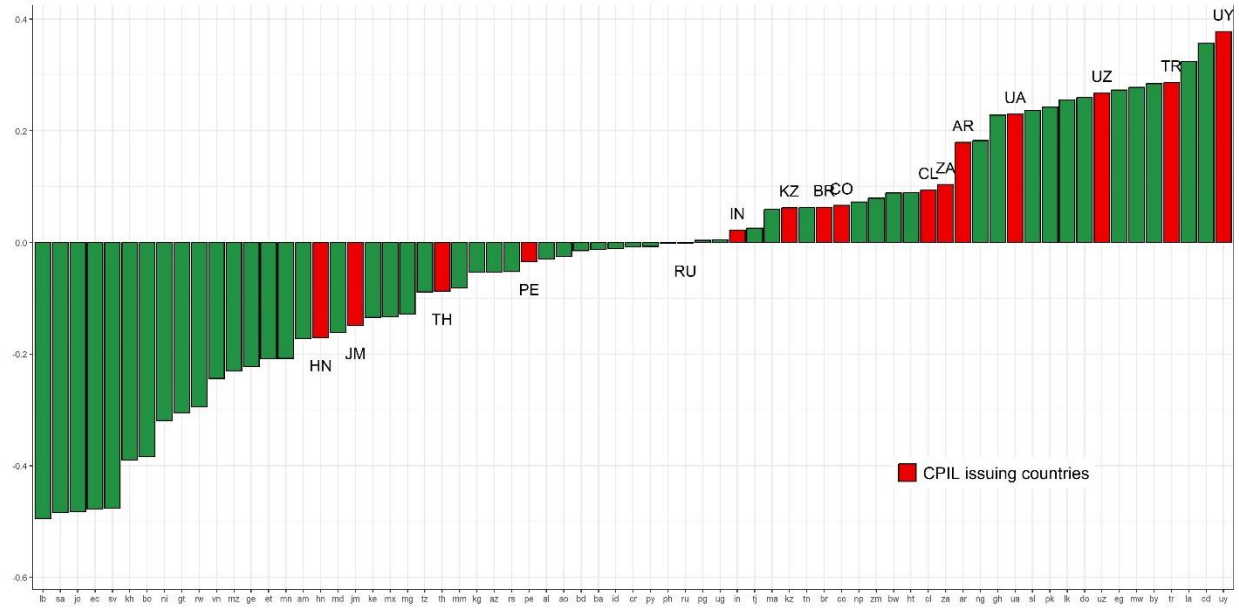
Source: Authors' calculations

Prediction 2: Countries with lower risk-adjusted costs of CPIL borrowing (higher CPIL SRs) relative to FCY or LCY borrowing are more likely to issue CPIL bonds than otherwise.

In checking for the effect of risk tolerance on the CPIL issuance we use the excess returns and the corresponding standard errors in computing ex-post Sharpe ratios for all three forms of investment.[18] First, we compute the borrower's ex-post excess returns for a given investor's SR for three different horizons: one month, one year, and three years. The results for all countries are shown in the appendix. To arrive at the borrower's SR, we then combine these excess returns with the ex-post standard errors of these instruments computed from macroeconomic indicators on the relevant horizons. For illustration, the results for the investor's SR of 0.5 and 12M horizon are displayed in Figure 5.

[18] It is useful to bear in mind that these are hypothetical SRs computed using historical macroeconomic data and not (i) actual SRs achieved on specific investments over the period, (ii) SRs that would have been expected to be achieved by investors when making the investment decisions during this period.

Figure 5. Calculated borrower's Sharpe ratio corresponding to investor's Sharpe ratio = 0.5 (12-month horizon)



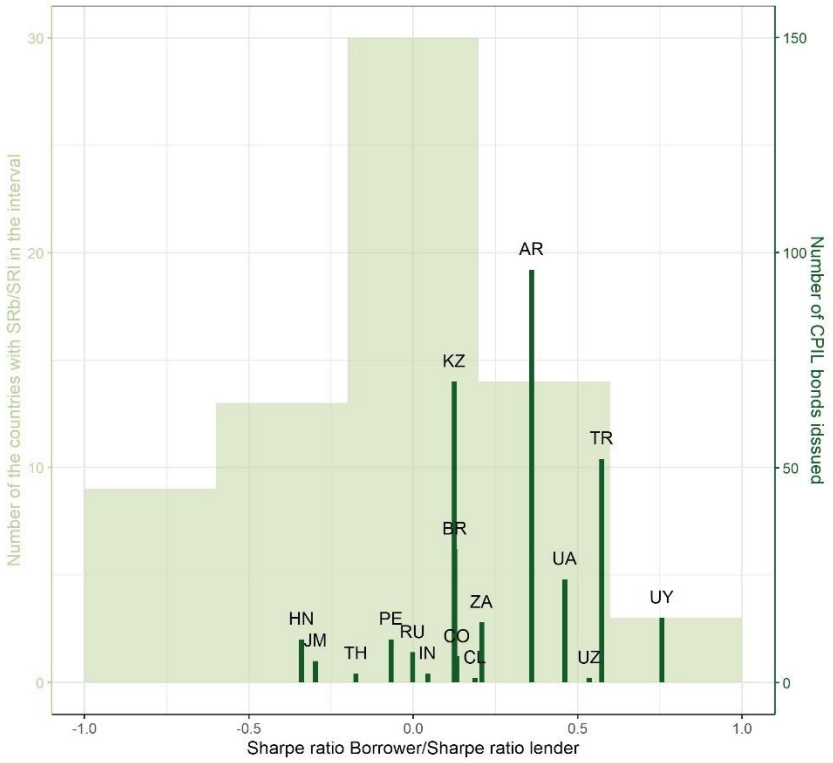
Source: Authors' calculations

We see in Figure 5 that, with the exception of the 4 countries above (Jamaica, Honduras, Peru, and Thailand), all CPIL issuing countries had a positive ex-post SR, implying that their risk-unadjusted costs from borrowing in CPIL relative to borrowing in LCY would have been lower. We also see that the CPIL issuing countries would have benefitted from the CPIL borrowing more than the rest of the sample and the difference is statistically significant.[19] We therefore see some evidence that CPIL borrowing is more likely to occur in countries with higher CPIL Sharpe ratios.

To see how these high SRs from CPIL borrowing compare with the SRs of the FCY alternative, we then compute a ratio between this SR of the CPIL borrower and the SR of the borrower in FCY.[20] In Figure 6, we order the countries by the size of this ratio and find that most CPIL issuing economies have this ratio relatively high, significantly higher than the rest of the sample (Figure 6).[21] Similar charts for the 1M and 36M horizons are in the appendix. We therefore have some evidence that the countries with lower risk-adjusted costs of CPIL borrowing (higher CPIL SRs) relative to the FCY borrowing are more likely to issue CPIL bonds than otherwise. In other words, the Sharpe ratio optic may indeed matter for the incidence of CPIL borrowing.

[19] The sample average SR for the countries that did not issue CPIL is negative, -0.06, while the average for CPIL-issuing countries equals 0.08, and increases to 0.15 when removing four countries with negative SRs.
[20] Note that the SR of the borrower in FCY is the same as that of the investor in CPIL or LCY by construction.
[21] The sample average SR for the countries that did not issue linkers is negative, -0.12, while the average for CPIL-issuing countries equals 0.16, and increases to 0.29 when removing four countries with negative SR.

Figure 6. Histogram of relative ex-post SRs between the borrower and lender on the 12-month horizon



Source: Authors' calculations

Prediction 3: The risk-adjusted costs of CPIL instruments will always be higher than those of conventional FCY alternatives.

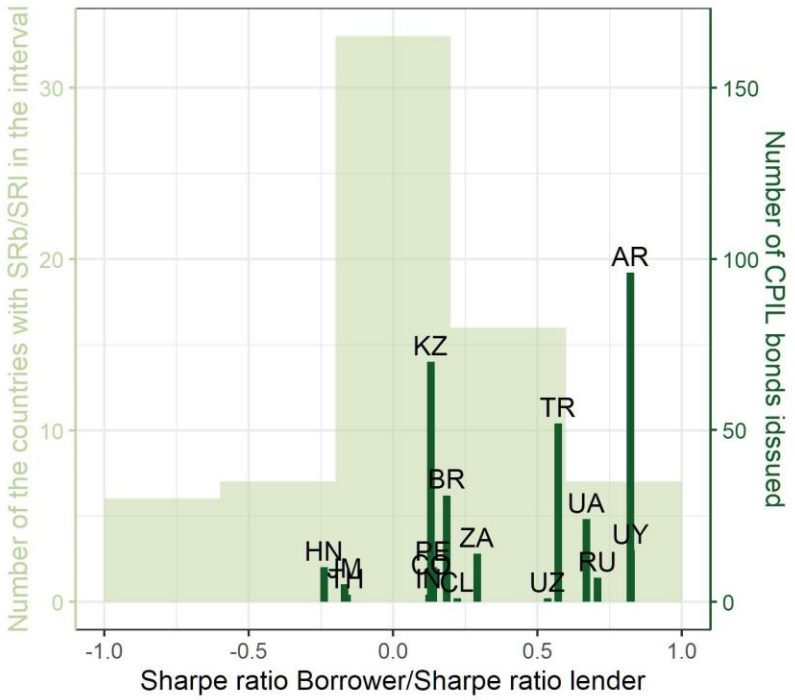
We saw in Figure 5 and Figure 6 evidence that the Sharpe ratio optic matters for the CPIL issuance: the countries with higher CPIL SRs are more likely to issue CPIL bonds than otherwise. However, this does not mean that the CPIL option is cheaper than the FCY option after adjusting for risks.

In fact, we see in Figure 6 that, as expected, the ratios of the SRs of a CPIL borrower and FCY borrower were in all cases lower than one for all countries (not only those issuing CPIL) for the 12M horizon. Figure 7 completes this information by showing a histogram of the maximum values of these ratios across all three horizons. Again, the same pattern emerges. This means that the risk-adjusted costs of the CPIL borrowing were on average larger than those of FCY borrowing.

Finding 4: Risk-averse borrowers will never find the CPIL instruments as attractive as risk-averse foreign investors

At the same time, Figure 6 and Figure 7 imply that the SR of the CPIL borrower has always been lower than the SR of the CPIL investor, creating a risk-return wedge between the two. This demonstrates in practice our theoretical assertion that the CPIL borrower can never be compensated for the risk as well as the CPIL investor.

Figure 7. Histogram of maximum relative ex-post SRs between the borrower and lender across the 1M, 12M and 36M horizons



Source: Authors' calculations

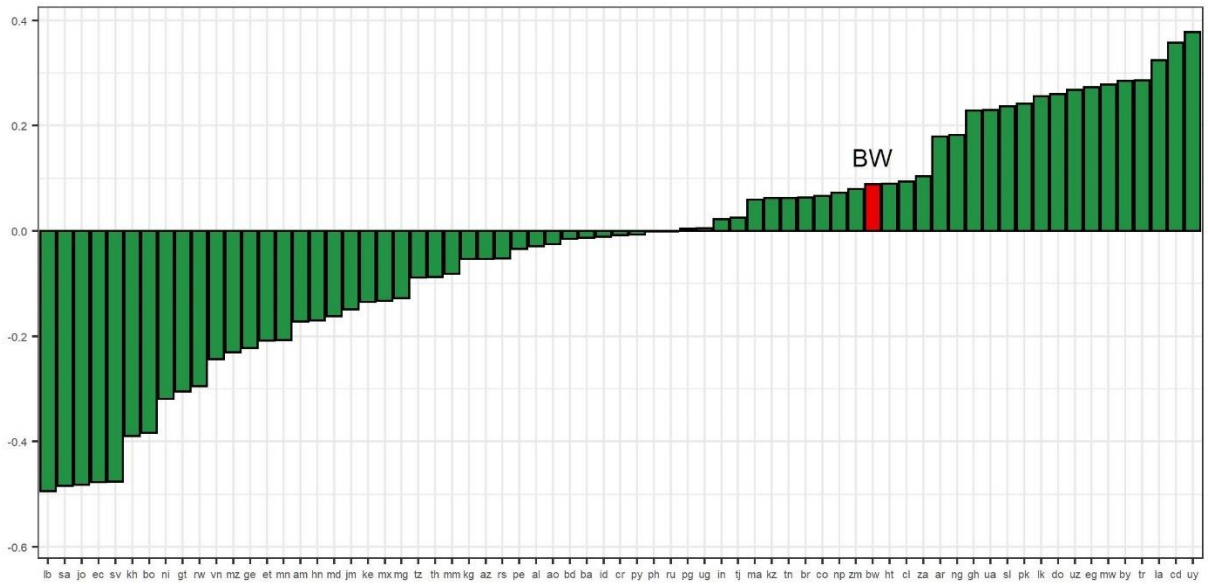
E. Country Application

This section presents a practical application of our approach in identifying good cases for CPIL bonds in practice. As announced in the 2024/2025 Budget Speech, the Botswana Government continues to run budget deficits while pursuing the reform agenda outlined in the Transitional National Development Plan. In this context, domestic borrowing will include the issuance of new instruments, such as inflation-linked bonds and green bonds, among others, contributing to the development of the capital market.

Botswana is a country that runs a crawling peg infused with elements of inflation targeting. Inflation has been under good control for decades. According to our methodology, Botswana is among the countries that stand to benefit from issuing CPIL bonds. Depending on the horizon, Condition 1 is satisfied, or at least there is a positive correlation between inflation and exchange rates, placing the country in either Case 1 or Case 2 of Exhibit 1.

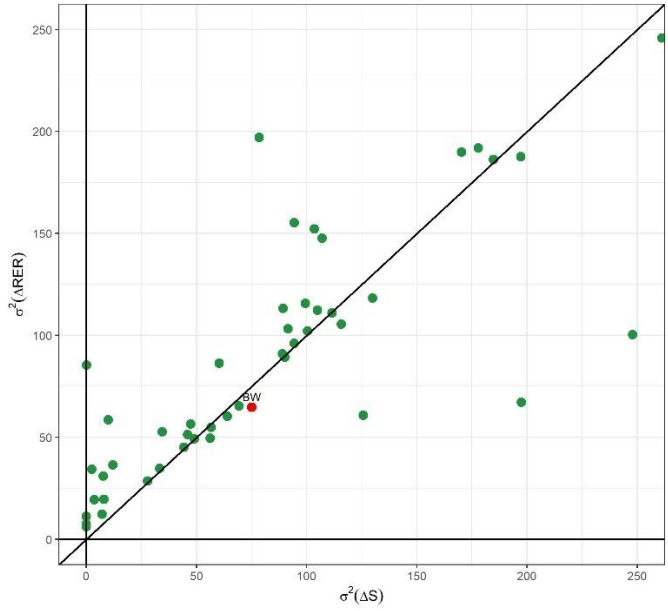
Historically, the excess return for lender and borrowers in Botswana has been quite high compared to other countries in our sample. These values are similar to those in Brazil, which was the fourth-largest CPI-linked bond issuer in our sample.

Figure 8. Borrower's Sharpe ratio corresponding to investor's Sharpe ratio = 0.5 (12-month horizon) - Botswana's relative position is highlighted



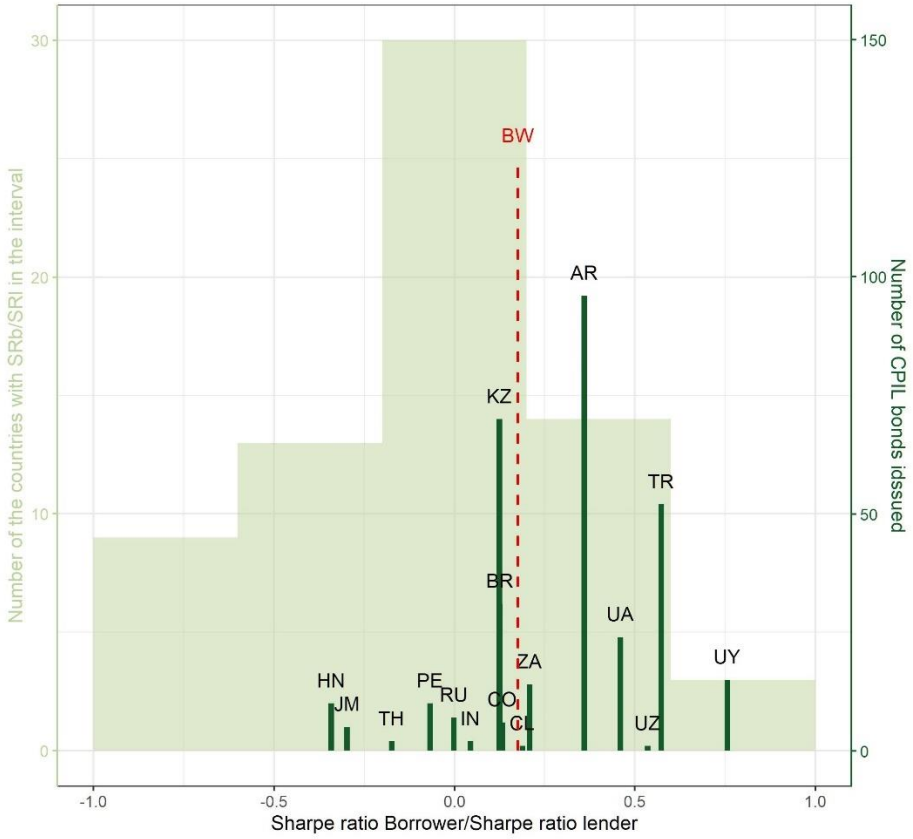
Source: Authors' calculations

Figure 9.Sample standard errors of real and nominal exchange rate for 12-month horizon - Botswana's relative position is highlighted



Source: Authors' calculations

Figure 10. Histogram of relative ex-post SRs between the borrower and lender on the 12-month horizon - Botswana's potential position is highlighted



Source: Authors' calculations

The introduction of CPI-linked bonds could therefore provide Botswana with a significant opportunity to manage its public debt efficiently. By aligning bond returns with inflation rates, these bonds can help attract investors seeking protection against inflation, ultimately supporting Botswana's broader economic objectives. Given the positive historical correlations and returns, Botswana's move towards CPI-linked bonds would not be much costlier than plain FCY borrowing, while at the same time it could serve as a strategic tool for enhancing fiscal resilience and market confidence.

F. Conclusions

Our research shows that the limited utilization of CPI-linked (CPIL) financing instruments in developing and emerging market economies may owe to inferior risk-return characteristics of these instruments compared to other financing alternatives, e.g., in FCY or plain LCY.

Limiting ourselves to the implications of the market risk only, we consider the risk-return perspective of a foreign investor as well as a domestic (sovereign) borrower. We show that the interest rates on LCY and CPIL instruments can be set so that the investor is indifferent between investing in LCY, CPIL or FCY. By contrast, the borrower will always tend to prefer FCY or plain LCY borrowing to CPIL instruments, because the latter has higher risk-adjusted costs.

Only a risk-neutral borrower may find CPIL instruments more attractive than FCY and plain LCY instruments, because the expected, risk-unadjusted, borrowing costs can under certain conditions be lower. These conditions require that the correlation between inflation and nominal depreciation is positive and high enough. The positive correlation offsets the impact depreciation volatility has on CPIL returns for the investor, resulting in lower market risk premiums required by the investor and therefore also lower borrowing costs than the plain LCY or FCY instruments.

This condition implies that the economies most likely to benefit from CPIL financing are those with (i) high nominal depreciation volatility relative to inflation volatility, and (ii) low real depreciation volatility. This is because CPIL instruments provide a hedge against business cycle volatility characterized by co-movements of inflation and depreciation, but not against shocks to the real exchange rate or terms-of-trade. Perhaps surprisingly, economies with good inflation control are better suited for CPIL instruments compared to those with high exchange rate control, ceteris paribus.

Considering a risk averse borrower brings more complex considerations. We show theoretically that the attractiveness of CPIL borrowing depends on its risk-adjusted costs and find empirically that countries with lower risk-adjusted costs of CPIL borrowing (higher CPIL SRs) relative to FCY or LCY borrowing have been more likely to issue CPIL bonds than otherwise.

Nevertheless, we also show that the risk-adjusted costs of CPIL instruments will always be higher than those of conventional FCY and LCY alternatives, thus making the CPIL instruments in general a less preferred way of borrowing. Furthermore, we find that the CPIL borrower can never be compensated for the market risk as well as the CPIL investor, putting a wedge between the incentives of both parties to engage in CPIL financing by making CPIL more attractive for the investor than the borrower.

Overall, we find robust empirical support for our assertion that the risk-return characteristics given by the macroeconomic environment are an important factor in determining the attractiveness of CPIL financing. Most CPIL-issuing economies have met the inflation-exchange rate correlation condition. Only one out of 16 CPIL-issuing economies in our sample did not satisfy these conditions at all. Moreover, the group of CPIL bond issuing countries was likely to achieve lower risk-adjusted costs of CPIL borrowing relative to FCY or LCY than the rest of the sample on average. Nevertheless, we also show that these risk-adjusted costs of CPIL borrowing would still have been higher than those of FCY borrowing or the risk-adjusted return achieved by the potential foreign investor.

Our research yields several important policy implications. One is that CPIL financing is unlikely to be a preferred way of borrowing in developing and emerging market economies irrespective of the ex-ante market risk expectations. This would require either a risk neutral-borrower or some channel through which CPIL instruments would reduce some other than market risk facing the investor (e.g., the credit risk) and thus lead to an overall cheaper source of financing. Furthermore, CPIL instruments are not suitable for economies with strong control over the exchange rate and weak control of inflation. Finally, they are not suitable for countries with large volatility in the real exchange rate and terms-of-trade.

More work is needed. We are planning to expand the analysis to more countries, including from developed economies. We would also like to consider and compare actual prices of LCY, FCY and CPIL instruments issued by governments in the past. This should give more insight into the actual Sharpe ratios that investors and borrowers have realized through these vehicles. We should also consider different maturities of the debt and analyze how the CPIL attractiveness may vary across different tenors. It will also be worthwhile

to extend the search for CPIL instruments through other data sources, potentially including private sector borrowers.

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Math Appendix

Definitions

Let i_t represent a 1-period interest rate. We will denote $i_{t,h}$ the average interest rate on the investment made h periods ago at time t.

$$i_{t,h} = \frac{1}{h}\sum_{j=0}^{h}(i_{t-j})$$

For simplifying notations, we will use the Δ_{h} operator to represent the annualized percent change of any variable X_t depending on the h period being discussed

$$\Delta_{\mathrm{h}} X_t \stackrel{\text{def}}{=} 100 \cdot \left(\left(\frac{X_t}{X_{t-h}} \right)^{\frac{12}{h}} - 1 \right)$$

In line with this definition, we will refer to inflation as the h -period annualized CPI growth: $\pi_t = \Delta_{\mathrm{h}} CPI_t$

We will apply the expectations operator at the time the investment is made (i.e., $t-h$). To simplify notations, we will omit the subscript: $E(\cdot) \stackrel{\text{def}}{=} E_{t-h}(\cdot)$.

In line with economic theory, we will use the Sharpe ratio to measure the compensation for risk. For any instrument $Y_{t,h}$ with the excess return $ER\left(Y_{t,h}\right)$, the Sharpe ratio will be defined as:

$$SR^{Y_{t,h}} = \frac{E\left(ER\left(Y_{t,h}\right)\right)}{\sigma\left(ER\left(Y_{t,h}\right)\right)}$$

Due to the market risk involved in various forms of investing into emerging market and developing economies, investors demand higher expected excess returns to remain indifferent among these different investment forms.

We postulate that the investor sets the lending interest rates so that the anticipated excess return from each form of investment should be adjusted to yield a Sharpe ratio equal to or greater than a predetermined positive threshold. This implies the following expression for the lending rate in local currency:

$$i_{t,h}{}^{LCY} = E\Delta_{\mathrm{h}} S_t + i^*_{t,h} + \lambda_t$$

The interest rate on CPI-linked (CPIL) investment can be analogously represented as some premium (ω_t) over the inflation rate:

$$i_{t,h}{}^{LCY,CPIL} = \pi_t + \omega_t$$

Foreign Investor Characteristics

Table 4. Risk-return characteristics of the different instruments for the investor

Investor:	FCY	LCY	CPIL
Lending interest rate	$i^*_{t,h}$	${i_{t,h}}^{LCY}$	${i_{t,h}}^{LCY,CPIL}$
Excess return	$ER^{Inv,FCY}_{t,h} = 0$	$ER^{Inv,LCY}_{t,h} = E\Delta_{\mathrm{h}}S_t + \lambda_t - \Delta_{\mathrm{h}}S_t$	$ER^{Inv,LCY,CPIL}_{t,h} = \pi_t + \omega_t - \Delta_{\mathrm{h}}S_t - i^*_{t,h}$
Expected Excess Return	$EER^{Inv,FCY}_{t,h} = 0$	$EER^{Inv,LCY}_{t,h} = \lambda_t$	$EER^{Inv,LCY,CPIL}_{t,h} = E\pi_t + \omega_t - E\Delta_{\mathrm{h}}S_t - i^*_{t,h}$
Standard error of the Excess return	$\sigma\left(ER^{Inv,FCY}_{t,h}\right) = 0$	$\sigma\left(ER^{Inv,LCY}_{t,h}\right) = \sigma^{\Delta_{\mathrm{h}}S}$	$\sigma\left(ER^{Inv,LCY,CPIL}_{t,h}\right) = \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_{\mathrm{h}}S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}}S}}$
Sharpe Ratio	NA	$SR^{Inv,LCY} = \frac{\lambda_t}{\sigma^{\Delta_{\mathrm{h}}S}}$	$SR^{Inv,LCY,CPIL} = \frac{E\pi_t + \omega_t - E\Delta_{\mathrm{h}}S_t - i^*_{t,h}}{\sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_{\mathrm{h}}S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}}S}}}$

Source: Authors' calculations

Investing in FCY:

Investing in FCY is the risk-free investment for the investor at the rate of $i^*_{t,h}$.

Investing in LCY:

For the foreign investor who is investing in the LCY bonds, the excess return will be:

$$ER^{Inv,LCY}_{t,h} = {i_{t,h}}^{LCY} - E\Delta_{\mathrm{h}}S_t - i^*_{t,h} = E\Delta_{\mathrm{h}}S_t + \lambda_t - \Delta_{\mathrm{h}}S_t$$

Expected excess returns and their standard deviation are:

$$EER^{Inv,LCY}_{t,h} = E\Delta_{\mathrm{h}}S_t + \lambda_t - E\Delta_{\mathrm{h}}S_t = \lambda_t \; ; \sigma\left(ER^{Inv,LCY}_{t,h}\right) = \sigma^{\Delta_{\mathrm{h}}S}$$

The Sharpe ratio will then be:

$$SR^{Inv,LCY} = \frac{\lambda_t}{\sigma^{\Delta_{\mathrm{h}}S}}$$

So, for given $SR = \overline{SR}$, λ_t can be calculated as $\lambda_t = \overline{SR} \cdot \sigma^{\Delta_{\mathrm{h}}S}$

Plugging it back, gives the following expression for the excess return and its expectations:

$$ER^{Inv,LCY}_{t,h} = E\Delta_{\mathrm{h}}S_t + \overline{SR} \cdot \sigma^{\Delta_{\mathrm{h}}S} - \Delta_{\mathrm{h}}S_t$$
$$EER^{Inv,LCY}_{t,h} = \overline{SR} \cdot \sigma^{\Delta_{\mathrm{h}}S}$$

Investing in CPIL:

Excess return of the offshore investor in LCY CPIL.

$$ER_{t,h}^{Inv,LCY,CPIL} = {i_{t,h}}^{LCY,CPIL} - \Delta_{\mathrm{h}} S_t - i_{t,h}^* = \pi_t + \omega_t - \Delta_{\mathrm{h}} S_t - i_{t,h}^*$$

Expected excess returns and their variance are:

$$EER_{t,h}^{Inv,LCY,CPIL} = E\pi_t + \omega_t - E\Delta_{\mathrm{h}} S_t - i_{t,h}^*;\ \sigma^2\left(ER_{t,h}^{Inv,LCY,CPIL}\right) = (\sigma^{\pi})^2 + \left(\sigma^{\Delta_{\mathrm{h}} S}\right)^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}} S}$$

Sharpe ratio is defined as $SR^{Inv,LCY,CPIL} = \frac{E\pi_t + \omega_t - E\Delta_{\mathrm{h}} S_t - i_{t,h}^*}{\sqrt{(\sigma^{\pi})^2 + \left(\sigma^{\Delta_{\mathrm{h}} S}\right)^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}} S}}}$

So, for a given value of SR of $\overline{SR}$, ω_t can be calculated as $\omega_t = \overline{SR} \cdot \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_{\mathrm{h}} S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}} S}} - E\pi_t + E\Delta_{\mathrm{h}} S_t + i_{t,h}^*$

Plugging it back, gives the following expression for the excess return and its expectations:

$$ER_{t,h}^{Inv,LCY,CPIL} = \pi_t - \Delta_{\mathrm{h}} S_t + \overline{SR} \cdot \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_{\mathrm{h}} S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}} S}} - E\pi_t + E\Delta_{\mathrm{h}} S_t$$
$$EER_{t,h}^{Inv,LCY,CPIL} = \overline{SR} \cdot \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_{\mathrm{h}} S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}} S}} - E\pi_t$$

Domestic Borrower Characteristics

Table 5. Risk-return characteristics of the different instruments for the borrower

Borrower:	FCY	LCY	CPIL
Borrowing interest rate	$i_{t,h}^*$	${i_{t,h}}^{LCY}$	${i_{t,h}}^{LCY,CPIL}$
Excess return	$ER_{t,h}^{Bor,FCY} = {i_{t,h}}^{LCY} - i_{t,h}^* - \Delta_{\mathrm{h}} S_t = E\Delta_{\mathrm{h}} S_t + \lambda_t - \Delta_{\mathrm{h}} S_t$	$ER_{t,h}^{Bor,LCY} = 0$	$ER_{t,h}^{Bor,CPIL} = {i_{t,h}}^{LCY} - {i_{t,h}}^{LCY,CPIL} = E\Delta_{\mathrm{h}} S_t + i_{t,h}^* + \lambda_t - \pi_t - \omega_t$
Expected Excess Return	$EER_{t,h}^{Bor,FCY} = \lambda_t$	$EER_{t,h}^{Bor,LCY} = 0$	$EER_{t,h}^{Bor,CPIL} = E\Delta_{\mathrm{h}} S_t + i_{t,h}^* + \lambda_t - E\pi_t - \omega_t$
Standard error of the Excess return	$\sigma\left(ER_{t,h}^{Bor,FCY}\right) = \sigma^{\Delta_{\mathrm{h}} S}$	$\sigma\left(ER_{t,h}^{Bor,LCY}\right) = 0$	$\sigma\left(ER_{t,h}^{Bor,CPIL}\right) = \sigma^{\pi}$
Sharpe Ratio	$SR^{Bor,FCY} = \frac{\lambda_t}{\sigma^{\Delta_{\mathrm{h}} S}}$	NA	$SR^{Bor,CPIL} = \frac{E\Delta_{\mathrm{h}} S_t + i_{t,h}^* + \lambda_t - \pi_t - \omega_t}{\sigma^{\pi}}$

Source: Authors' calculations

Borrowing through LCY:

Borrowing through LCY is the risk-free borrowing for the borrower at the rate ${i_{t,h}}^{LCY}$.

Borrowing through CPIL:

Let us introduce the notion of the excess return for the borrower:

$$ER_{t,h}^{Bor,CPIL} = {i_{t,h}}^{LCY} - {i_{t,h}}^{LCY,CPIL}$$

Onboarding the UIP condition and the definition of ${i_{t,h}}^{LCY}$

$$ER_{t,h}^{Bor,CPIL} = {i_{t,h}}^{LCY} - {i_{t,h}}^{LCY,CPIL} = E\Delta_{\mathrm{h}} S_t + i_{t,h}^{*} + \lambda_t - \pi_t - \omega_t$$

Expected excess returns and its variance are:

$$EER_{t,h}^{Bor,CPIL} = E\Delta_{\mathrm{h}} S_t + i_{t,h}^{*} + \lambda_t - E\pi_t - \omega_t; \sigma^2\left(ER_{t,h}^{Bor,CPIL}\right) = (\sigma^{\pi})^2$$

Then the Sharpe ratio of the borrower will be

$$SR^{Bor,CPIL} = \frac{EER_{t,h}^{Bor,CPIL}}{\sigma\left(ER_{t,h}^{Bor,CPIL}\right)} = \frac{E\Delta_{\mathrm{h}} S_t + i_{t,h}^{*} + \lambda_t - \pi_t - \omega_t}{\sigma^{\pi}}$$

Borrowing through FCY:

If we define borrower's excess return for borrowing in FCY compared to the borrowing in LCY currency, we obtain:

$$ER_{t,h}^{Bor,FCY} = {i_{t,h}}^{LCY} - i_{t,h}^{*} - \Delta_{\mathrm{h}} S_t$$

Onboarding the definition of ${i_{t,h}}^{LCY}$

$$ER_{t,h}^{Bor,FCY} = E\Delta_{\mathrm{h}} S_t + i_{t,h}^{*} + \lambda_t - i_{t,h}^{*} - \Delta_{\mathrm{h}} S_t = E\Delta_{\mathrm{h}} S_t + \lambda_t - \Delta_{\mathrm{h}} S_t$$

Expected excess returns and its variance are:

$$EER_{t,h}^{Bor,FCY} = \lambda_t; \sigma^2\left(ER_{t,h}^{Bor,FCY}\right) = \left(\sigma^{\Delta_{\mathrm{h}} S}\right)^2$$

Then the Sharpe ratio of the borrower will be

$$SR^{Bor,FCY} = \frac{\lambda_t}{\sigma^{\Delta_h S}}$$

Proposition and Proofs

Proposition 1: if the excess return for the CPIL borrower is positive, borrowing in CPIL is cheaper relative to LCY borrowing:

$$EER_{t,h}^{Bor,CPIL} > 0 \Rightarrow EER_{t,h}^{Bor,CPIL} > EER_{t,h}^{Bor,LCY}$$

Proof:

The CPIL borrowing will be cheaper than plain LCY borrowing in expectations for the borrower, if

$$i_{t,h}{}^{LCY,CPI} < i_{t,h}{}^{LCY}$$

$$E\pi_t + \omega_t < E\Delta_h S_t + i_{t,h}^* + \lambda_t$$

$$E\pi_t + \omega_t - E\Delta_h S_t - i_{t,h}^* < \lambda_t$$

$$E\Delta_h S_t + i_{t,h}^* + \lambda_t - E\pi_t - \omega_t > 0$$

The last expression is equivalent to the $EER_{t,h}^{Bor,CPIL} > 0$. ■

Proposition 2: if the excess return for the CPIL borrower is positive, then it is also higher than the expected return from FCY borrowing:

$$EER_{t,h}^{Bor,CPIL} > 0 \Rightarrow EER_{t,h}^{Bor,CPIL} > EER_{t,h}^{Bor,FCY}$$

Proof:

From Proposition 1 it follows, that if the excess return for the CPIL borrower is positive, then

$$E\pi_t + \omega_t - E\Delta_h S_t - i_{t,h}^* < \lambda_t$$

If CPIL is cheaper for the borrower than plain LCY, then CPIL is also cheaper than FCY, because

$$-E\pi_t - \omega_t + E\Delta_h S_t + i_{t,h}^* > \lambda_t > 0$$
$$EER_{t,h}^{Bor,CPIL} > EER_{t,h}^{Bor,FCY} \blacksquare$$

Proposition 3: CPIL is cheaper relative to LCY borrowing if either of the following condition holds:

$$\rho > \frac{1}{2}\frac{\sigma^{\pi}}{\sigma^{\Delta_h S}}$$
$$\left(\sigma^{\Delta_h S}\right)^2 > \left(\sigma^{\Delta Z}\right)^2$$

Proof:

From Propositions 1 and 2 follows, that CPIL borrowing is cheaper relative to both LCY and FCY borrowing if $EER_{t,h}^{Bor,CPIL} > 0$. Using the expression for the expected excess return from Corollary 1 gives

$$EER_{t,h}^{Bor,CPIL} = \overline{SR} \cdot \left(\sigma^{\Delta_h S} - \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}}\right) > 0$$

$$\left(\sigma^{\Delta_h S}\right)^2 > (\sigma^{\pi})^2 + \left(\sigma^{\Delta_h S}\right)^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S} \equiv \left(\sigma^{\Delta Z}\right)^2$$

$$\left(\sigma^{\Delta_h S}\right)^2 > \left(\sigma^{\Delta Z}\right)^2$$

$$\left(\sigma^{\Delta_h S}\right)^2 + 2\rho\sigma^{\pi}\sigma^{\Delta_h S} > (\sigma^{\pi})^2 + \left(\sigma^{\Delta_h S}\right)^2$$

$$\rho > \frac{1}{2}\frac{\sigma^{\pi}}{\sigma^{\Delta_h S}} \blacksquare$$

Proposition 4: $SR^{Bor,CPIL} = \overline{SR}$ is possible only if the inflation and FX depreciation are perfectly correlated ($\rho = 1$):

Proof:

As derived above, the borrower's Sharpe ratio for the investment in CPIL bonds has the following expression:

$$SR^{Bor,CPIL} = \frac{E\Delta_h S_t + i_{t,h}^* + \lambda_t - \pi_t - \omega_t}{\sigma^{\pi}}$$

Plugging in the expressions for λ_t and ω_t from the investor's problem, we get the following relation between the Sharpe ratios of lender ($\overline{SR}$) and borrower ($SR^{Bor,CPIL}$):

$$\begin{aligned}\sigma^{\pi} \cdot SR^{Bor,CPIL} = {} & E\Delta_h S_t + i_{t,h}^* + \overline{SR} \cdot \sigma^{\Delta_h S} - E\pi_t - (\overline{SR} \cdot \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}} - E\pi_t \\ & + E\Delta_h S_t + i_{t,h}^*)\end{aligned}$$

$$\sigma^{\pi} \cdot SR^{Bor,CPIL} = \overline{SR} \cdot \sigma^{\Delta_h S} - \overline{SR} \cdot \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}}$$

$$SR^{Bor,CPIL} = \overline{SR} \cdot \frac{\sigma^{\Delta_h S} - \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}}}{\sigma^{\pi}}$$

Having $SR^{Bor,CPIL} = \overline{SR}$ in the expression above gives

$$\sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta S}} = \sigma^{\pi} - \sigma^{\Delta S}$$

$$(\sigma^{\pi})^2 + \left(\sigma^{\Delta S}\right)^2 - 2\rho\sigma^{\pi}\sigma^{\Delta S} = (\sigma^{\pi})^2 + \left(\sigma^{\Delta S}\right)^2 - 2\sigma^{\pi}\sigma^{\Delta S}$$

$$\rho = 1 \blacksquare$$

Proposition 5: $\frac{SR^{Bor,CPIL}}{\overline{SR}}$: increase with $\frac{\sigma^{\Delta_h S}}{\sigma^{\pi}}$

Proposition 6: $SR^{Bor,CPIL} \leq \overline{SR}$ for all values of $\frac{\sigma^{\Delta_h S}}{\sigma^{\pi}}$ and ρ:

Proof:

$$\frac{SR^{Bor,CPIL}}{\overline{SR}} = \frac{\sigma^{\Delta_h S}}{\sigma^{\pi}} - \sqrt{1 + \left(\frac{\sigma^{\Delta_h S}}{\sigma^{\pi}}\right)^2 - \frac{2\rho\sigma^{\Delta_h S}}{\sigma^{\pi}}} = k - \sqrt{1 + (k)^2 - 2\rho k}\,;\; k = \frac{\sigma^{\Delta_h S}}{\sigma^{\pi}}$$

Let us show, that $\frac{SR^{Bor,CPIL}}{\overline{SR}} \leq 1$ always holds

Let us denote. Then from the expression for Sharpe ratio we have:

$$k - \sqrt{1 + (k)^2 - 2\rho k} \leq 1$$

$$k \leq 1 + \sqrt{1 + (k)^2 - 2\rho k} \geq 1$$

$$k - 1 \leq \sqrt{(k-1)^2 + 2k(1-\rho)}$$

$\rho \in [-1; 1] => \; 2k(1-\rho) \geq 0$, which means that inequality always holds. ■

Corollary 1: Higher correlation between depreciation and inflation increases the borrower's risk-adjusted excess return from a CPIL bond

Proof:

As outlined earlier, from the prospective of a risk-averse investor with a positive Sharpe ratio $\overline{SR}$ (equal for all instruments),

$$\lambda_t = \overline{SR} \cdot \sigma^{\Delta_h S}$$

$$\omega_t = \overline{SR} \cdot \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}} - E\pi_t + E\Delta_h S_t + i^*_{t,h}$$

Plugging in the expressions for λ_t and ω_t into expected excess return for CPIL borrowing yields

$$EER^{Bor,CPIL}_{t,h} = E\Delta_h S_t + i^*_{t,h} + \overline{SR} \cdot \sigma^{\Delta_h S} - E\pi_t - \overline{SR} \cdot \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}} + E\pi_t - E\Delta_h S_t - i^*_{t,h}$$

$$EER_{t,h}^{Bor,CPIL} = \overline{SR} \cdot \left(\sigma^{\Delta_h S} - \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_h S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_h S}}\right)$$

Taking the derivative with respect to ρ

$$\frac{\partial EER_{t,h}^{Bor,CPIL}}{\partial \rho} = \overline{SR} \cdot \frac{\sigma^{\pi}\sigma^{\Delta S}}{2\sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta S}}} > 0 \blacksquare$$

Corollary 2*:* the risk adjusted costs of borrowing in CPIL cannot be lower than the risk-adjusted costs of borrowing in FCY

Proof:

In the case of local borrowing $SR^{Bor,FCY} = \overline{SR}$, as the expressions for $SR^{Bor,FCY}$ and $SR^{Inv,LCY}$ are the same (Tables 1 and 3).

Putting this together with Proposition 6 we obtain:

$$SR^{Bor,CPIL} \leq SR^{Bor,FCY} = SR^{Inv,CPIL} = SR^{Inv,LCY} = \overline{SR} \blacksquare$$

Corollary 3: If inflation expectations are unbiased (biased downward/biased upward), CPIL instruments will have higher expected excess returns than the FCY and LCY alternatives for both the investor and borrower, when nominal depreciation is equally (more/less) volatile than real depreciation

Proof:

To have a situation, when CPIL local borrowing is at the same time more profitable for investor and cheaper for borrower can be formally described as

$$\begin{cases} EER_{t,h}^{Inv,LCY,CPIL} \geq EER_{t,h}^{Inv,LCY} \\ EER_{t,h}^{Bor,CPIL} > 0 \end{cases}$$

Plugging in the expressions

$$\begin{cases} \pi_t + \omega_t - \Delta_h S_t - i_{t,h}^* \geq E\Delta_h S_t + \lambda_t - \Delta_h S_t \\ \pi_t + \omega_t \leq E\Delta_h S_t + i_{t,h}^* + \lambda_t \end{cases}$$

$$E\Delta_h S_t + \lambda_t + i_{t,h}^* \leq \pi_t + \omega_t \leq E\Delta_h S_t + i_{t,h}^* + \lambda_t$$

$$\pi_t + \omega_t = E\Delta_h S_t + i_{t,h}^* + \lambda_t$$

Plugging in the expressions for λ_t and ω_t for a fixed Sharpe ratio $\overline{SR}$

$$\pi_t + \overline{SR} \cdot \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_{\mathrm{h}}S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}}S}} - E\pi_t + E\Delta_{\mathrm{h}}S_t + i^*_{t,h} = E\Delta_{\mathrm{h}}S_t + i^*_{t,h} + \overline{SR} \cdot \sigma^{\Delta_{\mathrm{h}}S}$$

$$\pi_t + \overline{SR} \cdot \sqrt{(\sigma^{\pi})^2 + (\sigma^{\Delta_{\mathrm{h}}S})^2 - 2\rho\sigma^{\pi}\sigma^{\Delta_{\mathrm{h}}S}} - E\pi_t = \overline{SR} \cdot \sigma^{\Delta_{\mathrm{h}}S}$$

$$\pi_t + \overline{SR} \cdot \sigma^{\Delta Z} - E\pi_t = \overline{SR} \cdot \sigma^{\Delta_{\mathrm{h}}S}$$

$$\pi - E\pi + \overline{SR} \cdot (\sigma^{\Delta Z} - \sigma^{\Delta S}) = 0$$
$$\pi - E\pi = \overline{SR} \cdot (\sigma^{\Delta S} - \sigma^{\Delta Z})$$

Inflation expectations are unbiased, if $E\pi = \pi$, biased downward, if $E\pi < \pi$, and upward, if $E\pi > \pi$.

It follows that that if inflation expectations are unbiased, biased downward, biased upward, the CPIL instruments will have higher expected excess returns than the FCY and LCY alternatives for both the investor and borrower, if

$$\sigma^{\Delta S} = \sigma^{\Delta Z}, \sigma^{\Delta S} > \sigma^{\Delta Z}, \sigma^{\Delta S} < \sigma^{\Delta Z}$$

Correlation Analysis

Proposition 7: When inflation and nominal exchange rate depreciation are negatively correlated, then real exchange rate changes adjusted by foreign inflation are negatively correlated with domestic inflation.

Proposition 8: When real exchange rate changes adjusted by foreign inflation are positively correlated with domestic inflation, then inflation and nominal exchange rate depreciation are positively correlated

Corollary 4: Correlation between inflation and nominal exchange rate depreciation cannot be negative, if at the same time correlation between real exchange rate changes adjusted by foreign inflation and domestic inflation is positive

Proof:

$$\overline{\Delta Z} \equiv \Delta Z - \pi^* = \Delta S - \pi$$

Where ΔZ refers to real exchange rate depreciation, and $\overline{\Delta Z}$ is the real exchange rate depreciation adjusted by foreign inflation, i.e., a change in the real value of a unit of foreign currency in terms of domestic goods.

$$Var\left(\overline{\Delta Z}\right) = Var(\Delta S) + Var(\pi) - 2cov(\Delta S, \pi)$$

if $cov(\Delta S, \pi) < 0 \Rightarrow Var\left(\overline{\Delta Z}\right) > Var(\Delta S) + Var(\pi)$ (*)

$$\Delta S = \overline{\Delta Z} + \pi$$
$$Var(\Delta S) = Var\left(\overline{\Delta Z}\right) + Var(\pi) + 2cov\left(\overline{\Delta Z}, \pi\right)$$
$$2cov\left(\overline{\Delta Z}, \pi\right) = Var(\Delta S) - Var\left(\overline{\Delta Z}\right) - Var(\pi)$$

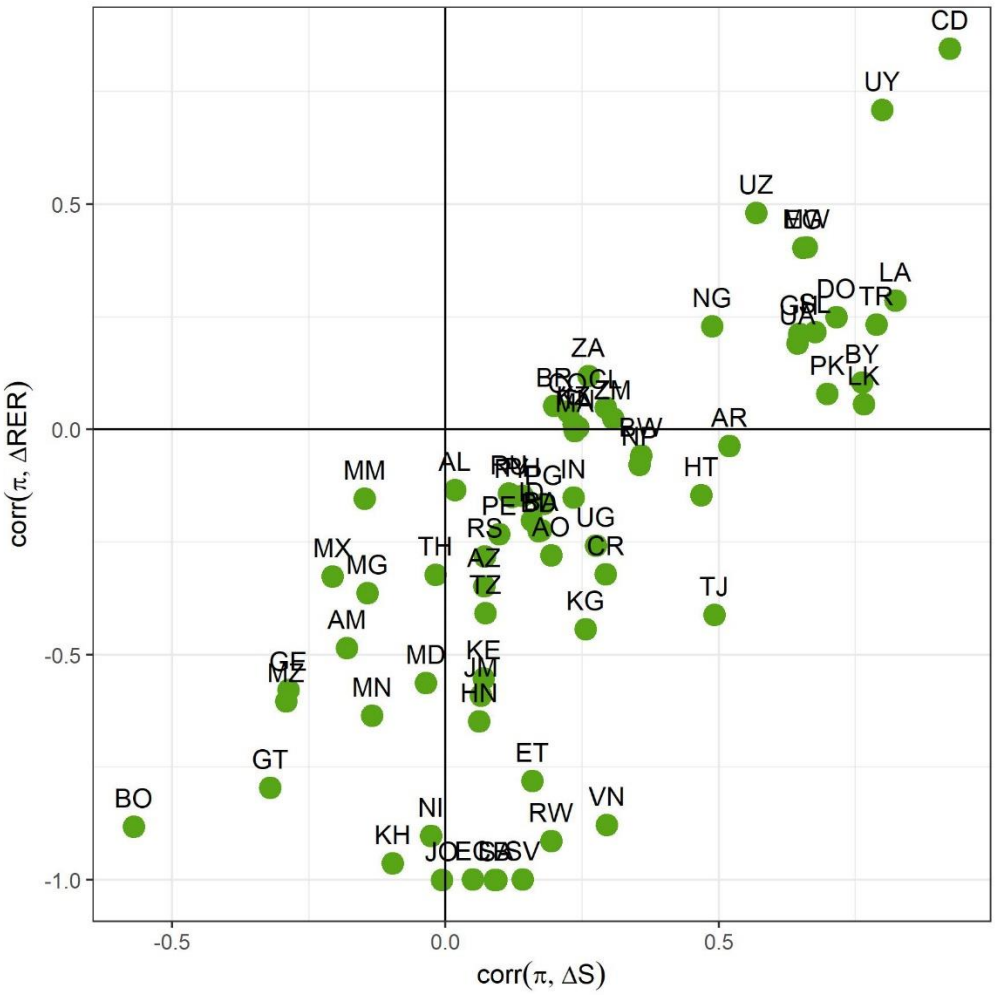
if $cov(\Delta S, \pi) < 0$, then from (*) $\Rightarrow cov\left(\overline{\Delta Z}, \pi\right) < 0$

if $cov\left(\overline{\Delta Z}, \pi\right) > 0 \Rightarrow Var(\Delta S) > Var\left(\overline{\Delta Z}\right) + Var(\pi)$

$$2cov(\Delta S, \pi) = Var(\Delta S) + Var(\pi) - Var\left(\overline{\Delta Z}\right) \Rightarrow cov(\Delta S, \pi) > 0$$

All possible combinations of correlation signs:

$cov(\Delta S, \pi)$	$cov\left(\overline{\Delta Z}, \pi\right)$
-	-
+	-
+	+



Source: Authors' calculations

Data Appendix

For empirical estimations, we utilized data on exchange rates and inflation for the following countries:

Albania, Angola, Argentina, Armenia, Azerbaijan, Bangladesh, Belarus, Bolivia, Bosnia and Herzegovina, Botswana, Brazil, Cambodia, Chile, Colombia, Congo Democratic Republic of, Costa Rica, Dominican Republic, Ecuador, Egypt, El Salvador, Ethiopia, Georgia, Ghana, Guatemala, Haiti, Honduras, India, Indonesia, Jamaica, Jordan, Kazakhstan, Kenya, Kyrgyz Republic, Lao People's Democratic Republic, Lebanon, Madagascar, Malawi, Mexico, Moldova, Mongolia, Morocco, Mozambique, Myanmar, Nepal, Nicaragua, Nigeria, Pakistan, Papua New Guinea, Paraguay, Peru, Philippines, Russian Federation, Rwanda, Saudi Arabia, Serbia, Sierra Leone, Sri Lanka, South Africa, Tajikistan, Tanzania, Thailand, Tunisia, Turkey, Uganda, Uruguay, Uzbekistan, Ukraine, Vietnam, Zambia.

The country-specific range for the excess return calculation is summarized in the table below.

al	04M08:23M12	*eg*	00M02:24M01	*mw*	00M02:24M01	*rs*	04M12:24M01
ao	02M11:24M01	*sv*	00M02:23M12	*mx*	00M02:24M01	*sl*	07M02:23M12
ar	00M02:23M11	*et*	04M03:23M11	*md*	00M02:24M01	*lk*	00M02:24M01
am	00M01:24M01	*ge*	01M06:24M01	*mn*	07M02:24M01	*za*	00M02:24M01
az	00M02:24M01	*gh*	00M02:24M01	*ma*	00M02:23M10	*tj*	00M01:23M12
bd	00M02:23M12	*gt*	00M02:24M01	*mz*	00M02:24M01	*tz*	00M02:24M01
by	00M02:24M01	*ht*	00M01:23M12	*mm*	00M02:22M12	*th*	00M02:24M01
bo	00M02:24M01	*hn*	00M02:23M11	*np*	00M02:23M12	*tn*	00M02:23M12
ba	00M07:23M10	*in*	00M02:24M01	*ni*	00M02:24M01	*tr*	00M02:24M01
bw	04M07:24M01	*id*	00M02:24M01	*ng*	00M02:24M01	*ug*	00M02:23M11
br	00M02:24M01	*jm*	00M02:23M12	*pk*	00M02:24M01	*uy*	00M02:23M12
kh	00M02:23M11	*jo*	00M02:24M01	*pg*	00M02:23M06	*uz*	11M05:22M09
cl	00M02:24M01	*kz*	00M02:24M01	*py*	00M02:24M01	*ua*	00M01:23M12
co	00M02:24M01	*ke*	00M02:24M01	*pe*	00M02:24M01	*vn*	00M02:24M01
cd	01M07:24M01	*kg*	00M01:24M01	*ph*	00M02:24M01	*zm*	00M02:24M01
cr	05M06:24M01	*la*	06M06:23M11	*ru*	00M02:23M12		
do	00M02:23M12	*lb*	00M02:21M10	*rw*	08M09:24M01		
ec	00M02:23M10	*mg*	04M11:23M11	*sa*	00M02:23M12		

We retrieved CPI-linked bond data from Cbonds. The table below summarizes the countries and the time spans during which they issued these bonds:

Country	*Timeframe of issuing CPI-linked bonds*
Jamaica	2/16/2010 - 2/22/2013
Honduras	11/26/2015-2/12/2021
Thailand	7/14/2011 - 3/12/2013
Brazil	1/8/2008-1/16/2024
India	04-06-13
South Africa	7/12/2013 - 9/15/2023
Peru	4/14/2004 - 8/21/2014
Uzbekistan	19-07-22
Colombia	2/25/2003 - 4/4/2015
Kazakhstan	1/27/2005 - 1/29/2016
Ukraine	06-10-17
Chile	28-02-13
Turkey	8/19/2008 - 2/20/2024
Uruguay	11/29/2003 -7/19/2023
Russia	7/17/2015 - 3/15/2023
Argentina	11/6/2001-9/30/2009 and 7/22/2016-3/12/2024

Source: Cbonds database

Empirical Appendix

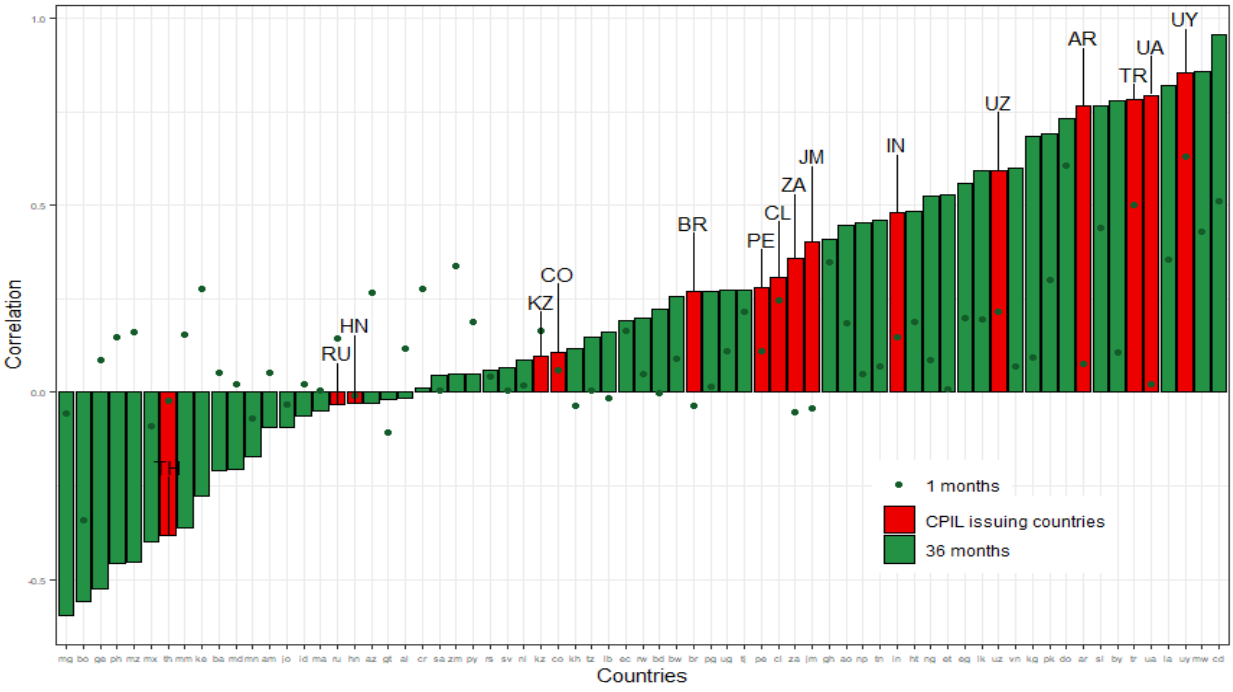
Calculated excess return for different horizons (Sharpe ratio = 0.5). Countries issuing CPIL papers are highlighted in blue.

	Median Investor Excess Return on CPIL			Median Investor Excess Return on LCY			Median Borrower Excess Return on CPIL		
	1M	12M	36M	1M	12M	36M	1M	12M	36M
al	17.97	5.64	2.66	18.24	5.79	2.49	0.46	0.41	0.19
ao	116.03	18.13	8.30	125.16	23.63	12.65	6.89	3.94	1.85
ar	3468.63	32.58	10.05	3541.04	48.34	11.08	75.32	20.90	13.34
am	24.61	2.81	0.79	25.19	3.85	1.58	1.73	-0.20	-0.30
az	61.58	7.27	6.77	66.74	10.40	8.02	4.68	1.64	0.20
bd	9.68	4.00	1.41	10.12	4.66	1.47	0.11	0.30	-0.03
by	453.87	15.50	4.44	469.47	26.41	13.17	16.04	13.26	8.39
bo	3.61	2.31	1.80	2.97	2.31	1.67	-1.46	-0.58	-1.06
ba	15.97	5.21	2.10	16.38	5.45	1.87	0.82	1.18	-0.03
bw	28.08	3.34	2.60	28.86	4.23	2.21	2.00	0.59	-0.74
br	58.24	12.41	4.35	57.32	12.93	4.82	0.56	0.59	0.49
kh	6.89	1.85	0.32	4.76	0.96	0.59	-2.42	-0.99	0.01
cl	28.78	6.64	2.95	30.09	7.01	3.53	0.95	0.96	0.64
co	41.36	8.46	4.42	41.82	8.45	4.54	0.78	0.70	-0.17
cd	1.8E+09	78.56	17.68	1.8E+09	140.32	34.13	497.67	54.20	14.16
cr	7.27	2.88	0.86	8.67	2.83	0.42	1.40	0.34	-0.05
do	34.85	6.68	2.84	44.51	14.44	7.10	8.94	6.17	3.54
ec	6.81	2.71	1.75	2.82	0.21	0.06	-4.01	-2.50	-1.69
eg	3776.63	17.05	6.18	3783.57	21.00	8.58	5.88	3.58	1.85
sv	2.28	0.56	0.95	0.42	0.06	0.02	-1.89	-0.55	-0.92
et	30.07	6.35	1.40	34.25	8.98	3.80	2.89	0.34	-0.27
ge	20.59	4.49	4.12	22.68	6.51	3.76	0.67	-0.71	-0.74
gh	59.67	10.76	3.49	68.18	17.63	6.57	7.83	5.27	1.53
gt	4.71	1.86	0.84	4.20	1.50	0.66	-0.58	-0.12	0.11
ht	34.27	6.09	2.54	41.08	11.31	3.44	3.44	2.58	1.36
hn	5.11	1.93	1.58	6.23	1.63	1.30	0.16	0.03	-0.20
in	12.77	3.37	0.97	15.16	4.34	1.93	-0.28	0.88	1.03
id	29.40	4.23	1.74	29.41	5.89	2.78	1.25	0.83	-0.05
jm	12.37	2.93	1.64	12.32	3.53	1.70	0.20	0.13	-0.06
jo	3.88	1.66	0.47	0.42	0.04	0.02	-3.38	-1.58	-0.44
kz	69.07	11.73	6.73	70.97	13.08	5.37	1.96	1.43	0.89
ke	11.89	3.67	2.21	14.05	5.53	2.09	1.41	1.08	0.45
kg	19.84	5.54	5.38	21.59	7.62	7.32	2.45	1.85	2.77
la	8.04	3.36	1.28	18.84	10.75	6.16	9.70	5.39	2.43

	Median Investor Excess Return on CPIL			Median Investor Excess Return on LCY			Median Borrower Excess Return on CPIL		
	1M	12M	36M	1M	12M	36M	1M	12M	36M
lb	33.93	7.85	3.53	0.81	0.19	0.05	-33.06	-7.62	-3.44
mg	17.94	4.13	2.66	19.16	4.77	2.13	1.01	-0.08	-0.50
mw	261.60	13.94	5.81	270.86	19.74	7.86	5.52	5.18	3.59
mx	39.63	7.25	2.94	39.64	7.00	2.78	0.07	0.10	0.02
md	16.40	5.88	4.25	15.98	4.45	3.79	1.14	0.35	0.12
mn	22.13	5.27	2.38	23.71	5.48	2.44	0.54	-0.85	-0.51
ma	14.10	4.29	1.78	14.01	4.29	2.02	-0.28	0.39	0.14
mz	36.88	16.73	9.13	38.54	13.56	6.92	2.66	-2.05	-3.24
mm	5.8E+25	1939.05	135.11	5.8E+25	1947.10	140.41	0.00	5.20	3.34
np	14.16	3.97	1.31	15.07	4.34	1.90	-1.14	-0.07	0.41
ni	6.35	1.45	0.89	5.30	0.83	0.40	-0.17	-0.66	-0.40
ng	138.64	12.95	4.16	142.82	15.87	5.13	2.49	1.96	0.57
pk	18.03	6.33	2.75	22.66	9.98	4.46	3.30	3.13	1.47
pg	16.12	4.49	1.77	17.76	6.38	2.75	0.44	0.57	0.53
py	27.23	7.88	2.96	27.52	8.85	3.60	1.76	1.36	0.29
pe	9.98	3.05	2.37	10.32	3.69	2.55	0.24	0.22	0.11
ph	10.71	3.28	1.69	11.00	3.47	1.69	0.56	0.29	-0.11
ru	169.14	14.12	5.47	170.70	14.56	6.45	4.93	0.81	-0.77
rw	7.86	1.69	0.44	7.04	2.18	0.89	-3.19	-1.15	0.25
sa	3.01	1.14	0.49	0.26	0.04	0.01	-2.79	-1.12	-0.48
rs	26.99	7.08	3.82	29.34	8.45	3.43	1.48	1.33	-0.26
sl	108.72	14.26	4.11	115.14	16.66	6.30	6.65	6.73	3.13
lk	63.13	4.55	2.09	72.43	11.25	4.16	8.27	6.23	1.92
za	54.60	6.89	4.04	53.93	6.90	4.33	0.24	0.50	0.29
tj	25.76	6.64	4.80	30.88	7.20	4.57	3.10	2.63	0.88
tz	13.67	5.05	2.56	15.39	5.76	2.20	1.28	0.71	0.74
th	12.66	4.00	2.48	12.20	3.81	2.58	-0.44	0.03	-0.43
tn	15.41	4.80	2.86	14.42	4.28	2.37	-0.01	0.22	0.45
tr	117.46	10.88	4.37	129.83	21.65	10.47	18.29	14.30	8.72
ug	19.39	4.50	3.24	20.06	6.43	3.91	1.54	0.98	0.88
uy	69.93	12.54	6.27	72.10	14.85	7.40	3.10	1.94	1.28
uz	7375.66	24.67	13.17	7378.08	29.45	18.67	1.61	1.15	-0.27
ua	983.35	15.13	8.65	988.44	22.82	14.04	6.88	5.24	3.31
vn	5.83	1.82	0.63	6.12	2.28	1.65	-0.09	0.41	0.96
zm	78.69	11.61	5.22	80.26	14.18	5.43	3.71	2.77	0.40

Appendix: Robustness analysis on different horizons

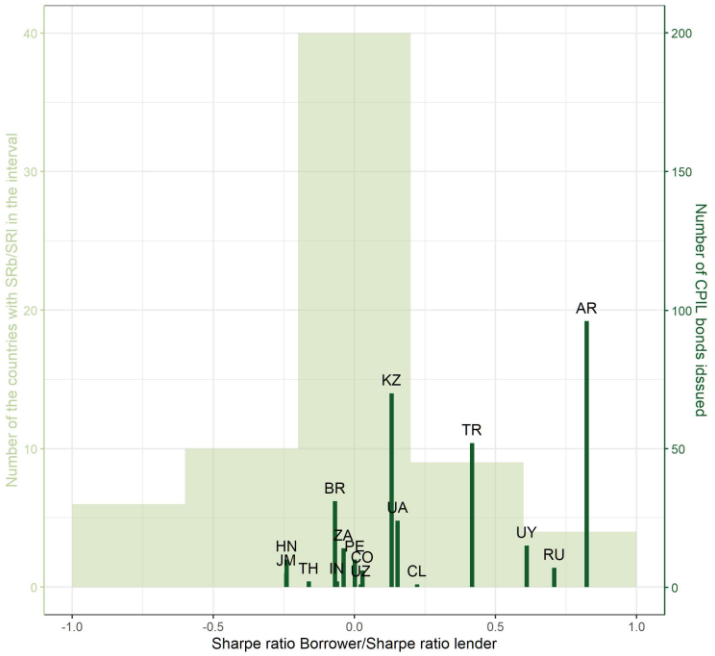
Correlation coefficients between inflation and changes in the nominal exchange rate (horizons 36 and 1 month)[22]



Source: Authors' calculations

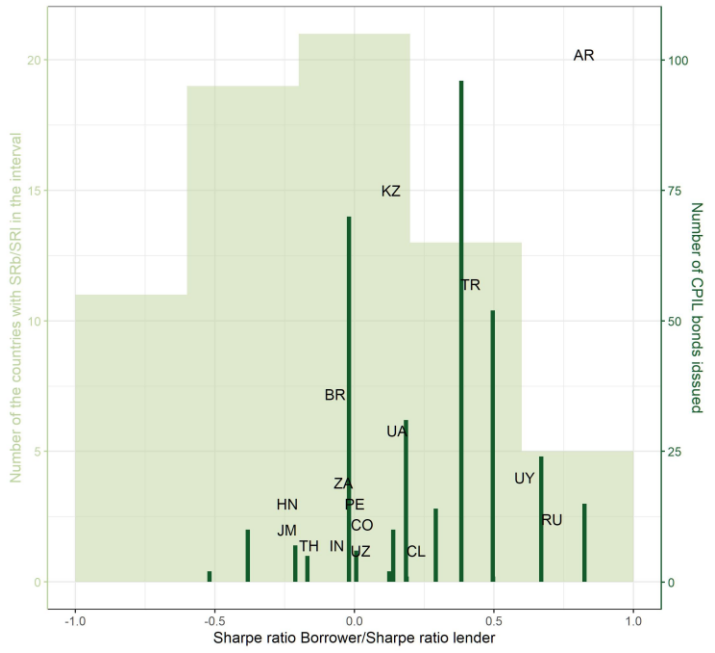
[22] Median correlation is 0.09 for the 1-month and 0.2 for the 36-month horizons

Issuance of CPIL instruments and their ex-post SR for 1-month horizon



Source: Authors' calculations

Issuance of CPIL instruments and their ex-post SR for the 36-month horizon



Source: Authors' calculations